Heaving and Pitching Airfoil

Per-Olof Persson and Chris Fidkowski

Overview

This problem is aimed at testing the accuracy and performance of high-order flow solvers for problems with deforming domains. A NACA 0012 airfoil is undergoing a smooth flapping-type motion, starting from rest at zero angle of attack and ending at a position one chord length higher. The metrics used to assess the accuracy of the solution are the total energy (i.e. integrated power) extracted from the flow during the motion and the vertical impulse imparted on the airfoil by the flow (integrated vertical force). Two different laminar Reynolds numbers are considered: $Re = 1000$ and $Re = 5000$.

Governing Equations

The governing equations for this problem are the 2D compressible Navier-Stokes equations with a constant ratio of specific heats equal to 1.4 and a Prandtl number of 0.72.

Geometry

The geometry consists of a NACA 0012 airfoil with chord length $c = 1$, with geometry modified to give zero trailing edge thickness:

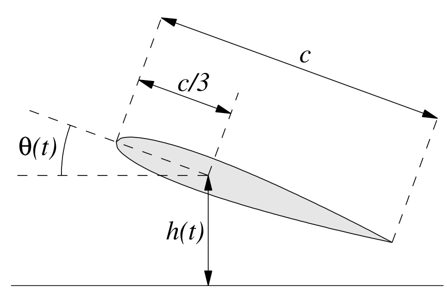
$$y(x) = \pm 0.6(0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4), \quad x \in [0, 1].$$

This undergoes a smooth upward motion of one chord length by heaving and pitching about a point located at the airfoil 1/3 chord location (see figure). The vertical displacement $h(t)$ and the pitching angle $\theta(t)$ are given by

$$h(t) = \begin{cases} 0, & \text{if } t < 0, \\ \frac{1-\cos \pi t}{2}, & \text{if } 0 \le t < 1, \\ 1, & \text{otherwise}, \end{cases} \tag{1}$$

and

$$\theta(t) = \begin{cases} 0, & \text{if } t < 0, \\ \frac{\pi}{6} \cdot \frac{1-\cos 2\pi t}{2}, & \text{if } 0 \le t < 1, \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$



The far-field boundary should be located at least 100 chord-lengths away from the airfoil.

Flow Conditions

The free-stream has a Mach number $M_\infty = 0.2$ and is horizontal, so that θ is the airfoil angle of attack. The Reynolds number based on the chord of the airfoil is $\text{Re} = 1000$ and 5000 for the two test cases. The initial condition at time $t = 0$ is the steady-state solution for the initial position $h = 0$, $\theta = 0$. To simplify post-processing, we assume convenient units in which the airfoil chord is $c = 1$ and the free-stream density and speed are unity, so that the free-stream conservative state vector is

$$[\rho, \rho u, \rho v, \rho E] = \left[1,\ 1,\ 0,\ 0.5{+}1/[M^2\gamma(\gamma-1)]\right]. \tag{3}$$

Output Quantities

The first output from the simulation is the work (energy) which the fluid exerts on the airfoil during the motion:

$$W = \int_0^1 \boldsymbol{F}(t) \cdot \boldsymbol{v}_0 \, dt + \int_0^1 \boldsymbol{T}(t) \cdot \boldsymbol{\omega} \, dt = \int_0^1 F_y(t)\dot{h}(t) \, dt + \int_0^1 T_z(t)\dot{\theta}(t) \, dt. \tag{4}$$

Here, $\boldsymbol{F}(t) = (F_x(t), F_y(t))$ is the force imparted by the fluid on the airfoil, $\boldsymbol{T}(t) = (0, 0, T_z(t))$ is the torque imparted by the fluid on the airfoil about the 1/3 chord pivot point, $\boldsymbol{v}_0 = \dot{h}(t)$ is the velocity of the pivot point, and $\boldsymbol{\omega}_0 = (0, 0, \dot{\theta})$ is the angular velocity of the airfoil about the pivot point.

The second output is the vertical impulse from the fluid onto the airfoil during the motion:

$$I = \int_0^1 F_y(t) \, dt \, . \tag{5}$$

Requirements

1. Perform the indicated simulation for the two test cases (a) $Re = 1000$ and (b) $Re = 5000$. Calculate the quantities W and I for the two cases, and perform a grid/timestep convergence study to get the values accurate to three digits (or as accurate as possible). Record the work units.

2. Provide the work units, the converged output values, nDOFs in the solution, and the distance to the far-field boundary for each case. Submit this data to the workshop contact.