C1.3 Laminar boundary layer on a flat plate

1. Code description

XFlow is a high-order discontinuous finite element library written in ANSI C, intended to be run on Linux-type platforms. XFlow supports DG and HDG discretizations and a variety of equation sets, including compressible Euler, Navier-Stokes, and RANS with the Spalart-Allmaras model. High-order is achieved compactly within elements using various high-order bases on triangles, tetrahedra, quadrilaterals, and hexahedra. Parallel runs are supported using domain partitioning and MPI communication. Visual post-processing is performed with an in-house plotter. Output-based adaptivity is available using discrete adjoints.

2. Case summary

The steady runs were performed using three discretizations:

- DG using second form of Bassi and Rebay viscous stabilization with $\eta_e = 2 \cdot \mathrm{nface}$,
- HDG using a constant viscous stabilization $\tau_v = 1$.
- Primal form of HDG using similar stabilization to that of DG.

Both discretizations used the following parameter set for all runs:

- Newton-GMRES solver with block incomplete lower upper factorization with no fill (block ILU0)
- 10 orders of magnitude L_1 residual convergence
- Roe approximate Riemann solver.

The runs were performed on a 8-core desktop with Intel(R) i7 CPUs and 16GB shared memory. On one core of this machine, one TauBench unit is equivalent to 6.866 seconds of compute time.

3. Meshes

XFlow could not converge on the first two meshes provided using either DG or HDG, so new meshes were generated for this case using the geometry given in the problem description and the following parameters:

- $L_V = 1.0$
- $L_H = 1.0$.

The new triangular and quadrilateral meshes are shown in Fig. 1. The coarsest triangular mesh has 280 elements and the coarsest quadrilateral mesh has 140 elements. Both these meshes have the first gridpoint at $y = 5.54 \cdot 10^{-5}$. The mesh sequence is created by coarsening the finest mesh.

4. Results

The truth value[1] was obtained by twice uniformly refining a hanging-node adapted mesh. This final mesh had 28560 quadrilateral elements and the solution was obtained with $p = 5$.

[1] $c_d = 1.31118365758 \times 10^{-3}$

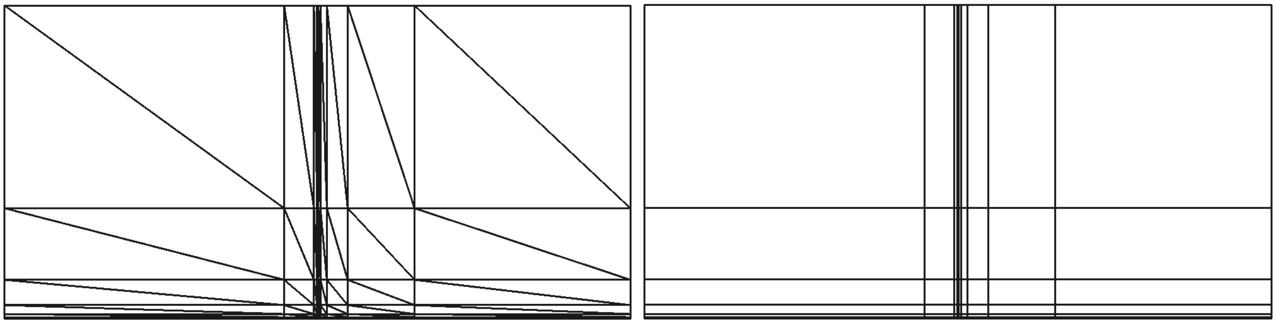

Figure 1: Coarsest level meshes.

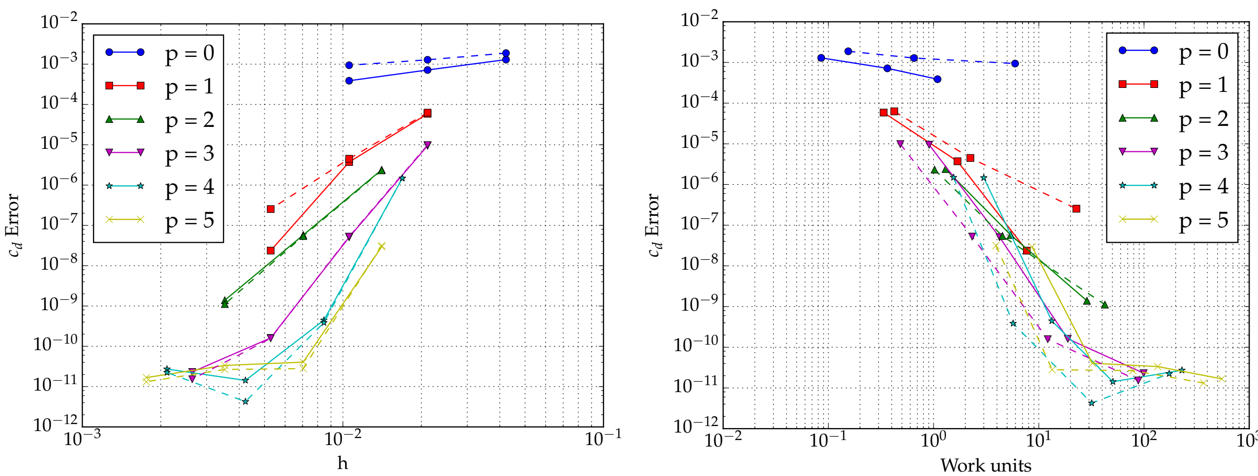

Figure 2: Drag convergence with relative mesh size (left) and work units (right). DG and primal HDG results are shown with solid and dashed lines respectively.

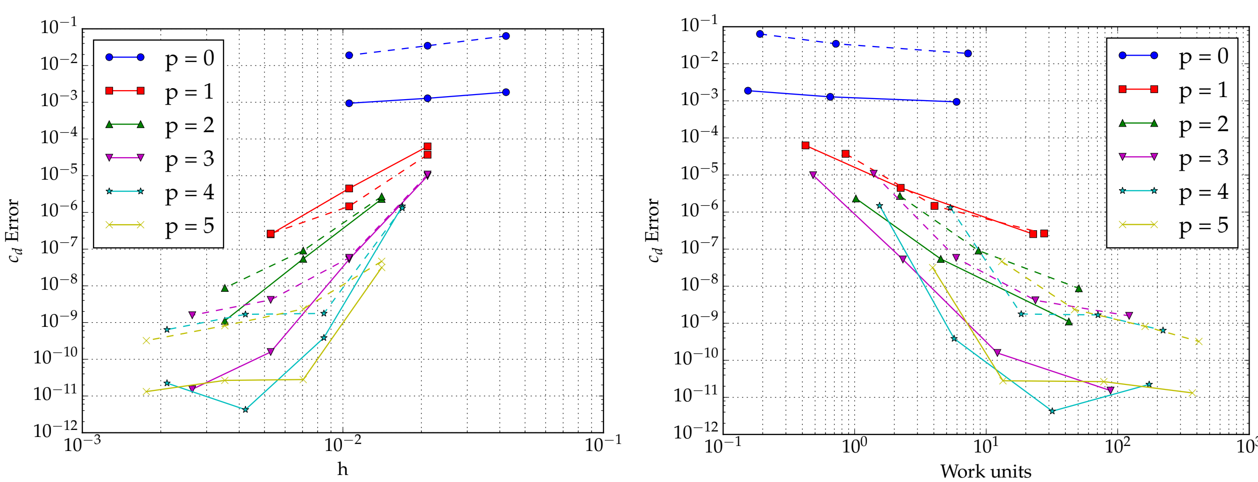

Figure 3: Drag convergence with relative mesh size (left) and work units (right). Primal and mixed HDG results are shown with solid and dashed lines respectively.

Order	ref 0	ref 1	ref 2	ref 3
0		$1.30 \cdot 10^{-3}$	$7.17 \cdot 10^{-4}$	$3.88 \cdot 10^{-4}$
1		$5.86 \cdot 10^{-5}$	$3.73 \cdot 10^{-6}$	$2.37 \cdot 10^{-8}$
2		$2.40 \cdot 10^{-6}$	$5.76 \cdot 10^{-8}$	$1.39 \cdot 10^{-9}$
3	$9.51 \cdot 10^{-6}$	$5.08 \cdot 10^{-8}$	$1.61 \cdot 10^{-10}$	$2.31 \cdot 10^{-11}$
4	$1.47 \cdot 10^{-6}$	$4.45 \cdot 10^{-10}$	$1.43 \cdot 10^{-11}$	$2.72 \cdot 10^{-11}$
5	$2.95 \cdot 10^{-8}$	$4.05 \cdot 10^{-11}$	$3.36 \cdot 10^{-11}$	$1.67 \cdot 10^{-11}$

(a) Errors

Order	ref 0	ref 1	ref 2
0		0.428	0.443
1		1.988	3.649
2		2.690	2.687
3	3.775	4.149	1.403
4	5.844	2.481	-0.466
5	4.756	0.135	0.502

(b) Rates

Table 1: Error and rates for DG results.

Order	ref 0	ref 1	ref 2	ref 3
0		$1.88 \cdot 10^{-3}$	$1.28 \cdot 10^{-3}$	$9.40 \cdot 10^{-4}$
1		$6.28 \cdot 10^{-5}$	$4.49 \cdot 10^{-6}$	$2.54 \cdot 10^{-7}$
2		$2.32 \cdot 10^{-6}$	$5.44 \cdot 10^{-8}$	$1.12 \cdot 10^{-9}$
3	$9.85 \cdot 10^{-6}$	$5.26 \cdot 10^{-8}$	$1.58 \cdot 10^{-10}$	$1.52 \cdot 10^{-11}$
4	$1.49 \cdot 10^{-6}$	$3.86 \cdot 10^{-10}$	$4.24 \cdot 10^{-12}$	$2.24 \cdot 10^{-11}$
5	$3.16 \cdot 10^{-8}$	$2.80 \cdot 10^{-11}$	$2.66 \cdot 10^{-11}$	$1.32 \cdot 10^{-11}$

(a) Errors

Order	ref 0	ref 1	ref 2
0		0.275	0.225
1		1.902	2.072
2		2.709	2.803
3	3.774	4.190	1.686
4	5.958	3.254	-1.201
5	5.069	0.038	0.506

(b) Rates

Table 2: Error and rates for primal HDG results.

Order	ref 0	ref 1	ref 2	ref 3
0		$6.39 \cdot 10^{-2}$	$3.46 \cdot 10^{-2}$	$1.90 \cdot 10^{-2}$
1		$3.73 \cdot 10^{-5}$	$1.46 \cdot 10^{-6}$	$2.64 \cdot 10^{-7}$
2		$2.72 \cdot 10^{-6}$	$9.15 \cdot 10^{-8}$	$8.77 \cdot 10^{-9}$
3	$1.08 \cdot 10^{-5}$	$5.81 \cdot 10^{-8}$	$4.15 \cdot 10^{-9}$	$1.58 \cdot 10^{-9}$
4	$1.32 \cdot 10^{-6}$	$1.78 \cdot 10^{-9}$	$1.68 \cdot 10^{-9}$	$6.42 \cdot 10^{-10}$
5	$4.65 \cdot 10^{-8}$	$2.35 \cdot 10^{-9}$	$8.23 \cdot 10^{-10}$	$3.22 \cdot 10^{-10}$

(a) Errors

Order	ref 0	ref 1	ref 2
0		0.441	0.432
1		2.336	1.235
2		2.448	1.691
3	3.768	1.903	0.696
4	4.767	0.042	0.692
5	2.154	0.755	0.677

(b) Rates

Table 3: Error and rates for HDG results.