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Abstract for the Onera NXO method

on case 1.1 : **Transonic Ringleb flow**

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1 Code description

Discretization by cell-centered Finite Volume Method, 1 dof / eqn / cell.

Upwind-biased convective scheme based on characteristic combination of the conservative variables extrapolated at the cell interface from either side, followed by an evaluation of the natural inviscid flux ("state-upwind" scheme, ref. 1).

Evaluation of left and right conservative variables states as surface averages on the interface, interpolated from the volume averaged conservative variables inside cells. The interpolations are based on weighted least-square polynomial reconstructions inside partially biased stencils (collection of cells centered on the left, respectively right cell on either side of the interface).

The least-square formulation presented in ref.1 is based on the set of high order moments of the cell metrics and interface metrics (volume integral and surface integral of each monomial of the polynomial reconstruction basis).
The Ringleb flow was computed using a recent development that takes into account high order geometry (the p4 geometry provided in gmsh format by the test-case coordinator).
The moments are computed by numerical cell and interface curvilinear integration (respectively 2D surface and 1D line curvilinear integrals in this 2D case).
A comparison is presented with the expression using linear elements.

The flux integration on each interface is also done at high order, as compared to the former version of the code where a single evaluation of the normal flux was done on each interface for each equation, from the interface average of the "state-upwinded" conservative fields. The interface is subdivided into elementary subfaces (from 2 to 5) with varying orientation of the surface vector, and the same polynomial reconstructed within each upside stencil is projected on each subface. The upwind characteristic scheme is then applied at each subface.
This procedure is used in the wall BC and at the open boundaries for high order extrapolation (for example pressure and normal velocity to a set of wall subfaces).

These combined reconstructions and projection phases are done in the pre-processor and provide sets of linear interpolation coefficients for the conservative variables fields and their gradients, from cell-averaged to surface-averaged quantities on each subface element.
The degree of the reconstructed polynomial is the highest enabled by the number of cells in the stencil (number of monomials *1.5), depending on the successive neighbours insertion (ref 1.).

Relevant solvers
Time accurate solutions either

- Non-linear implicit by dual time-stepping (Explicit RK for local pseudo-time stepping inner iterations), or
- Explicit Runge-Kutta options from 3 to 6 stages.

Explicit RK 4 stages was used for this test-case, CFL 0.25 with local pseudo-time stepping.

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Boundary conditions :

- outside boundary managed by Riemann invariants in the normal direction to the face, reference flow conditions are the exact analytical distribution of conservative variables on the inlet and outlet boundaries.
- slip boundary : standard formulation with compatibility relationships associated to the non-zero extrapolated normal velocity and imposing no through-flow fluxes. An attempt at imposing the exact values of the fluxes led to a code divergence.

High-order capability

k-exact reconstruction coded and verified up to a 6^{th} degree full base of monomials.
Polynomial reconstruction bases and stencil sizes are (k2– 13 cells), (k3 – 25 cells), (k4 and k5 – 41 cells).

Parallel capability

- Loop-based Open-MP programming.

For the runs of this test-case 2.3, an acceleration between 9 and 11 was recorded, on a dual-socket Westmere board (2 processors, 12 cores, 12 OMP threads) with respect to a single Westmere core.

Post-processing

- Output in TecPlot ™ Format.
- Internal binary format for visualizations inside the GUI.

2. **Case summary**

Machines used (number of cores if parallel) : 2-Westmere board, 12 cores, 12 OMP threads activated
Tau-bench wall clock times (sequential) on machines used : 7.8s.
The Work loads (in Tau_bench Work units) are computed as the wall clock time multiplied by 12, divided by the Tau-bench wall clock time (sequential).

3. **Meshes**

Description of meshes used for the case.

P4 Grids :

- grid3 and grid4 provided by the test-case coordinator, respectively 16*48 and 32*96 cells,
- grid5 obtained by interpolating the grid4 in the parameter space, generating new nodes for the high-order geometry on iso-parameter lines : 64*192 cells.

4. Results

The error is the computed L2 norm of the difference in the integral cell values of the conservative variables and the entropy with respect to the cell integrals of the exact fields.
Concerning the entropy, the reference cell averages are not constant in space, since they are computed from the cell averages of the analytical distribution of the conservative variables, as they are at the end of the computation on the converged fields.

In the post-processing, it is possible to compare the field distribution of the conservative and physical variables within the cells to the exact distribution. In each central cell of a reconstruction stencil, we plot the polynomial representation at high order (k2 to k5) of the conservative variables and the derived variables at each inner-cell interpolation node.

<u>Results on the 3 grids</u> (grid3 to grid5, p4 geometry, high order curvilinear moments and fluxes integration)

We compare the solutions for different reconstruction degrees : k2 to k4 in the field and at the wall (wider stencils are used at the boundaries to account for their unsymmetrical nature).
A grid convergence index is computed between 2 successive grids.

k2 for reconstruction, fluxes integration and BC

Grid	L2-Error(ro)	L2-Error(roU)	L2-Error(roV)	L2-Error(roEt)	L2-Error(s)
3 (16*48)	1.42e-3	3.31e-3	9.50e-4	2.71e-3	1.73e-3
cvg	***-2.07***	***-2.97***	***-2.02***	***-2.18***	***-2.24***
4 (32*96)	3.38e-4	4.20e-4	2.35e-4	6.03e-4	3.64e-4
cvg	***-2.00***	***-2.08***	***-1.85***	***-1.92***	***-1.71***
5 (64*192)	8.46e-5	9.90e-5	6.50e-5	1.59e-4	1.11e-4

The order of convergence of the k2 formulation is around 2.

k3 for reconstruction, fluxes integration and BC

Grid	L2-Error(ro)	L2-Error(roU)	L2-Error(roV)	L2-Error(roEt)	L2-Error(s)
3 (16*48)	1.23e-3	7.76e-4	5.27e-4	2.13e-3	5.80e-4
cvg	***-3.15***	***-3.18***	***-3.21***	***-3.14***	***-3.62***
4 (32*96)	1.39e-4	8.55e-5	5.68e-5	2.40e-4	4.70e-5
cvg	***-3.74***	***-3.92***	***-3.87***	***-3.72***	***-3.71***
5 (64*192)	1.04e-5	5.64e-6	3.88e-6	1.81e-5	3.58e-6

This k3 formulation approaches the theoretical order of convergence of 4.

k4 for reconstruction, fluxes integration and BC

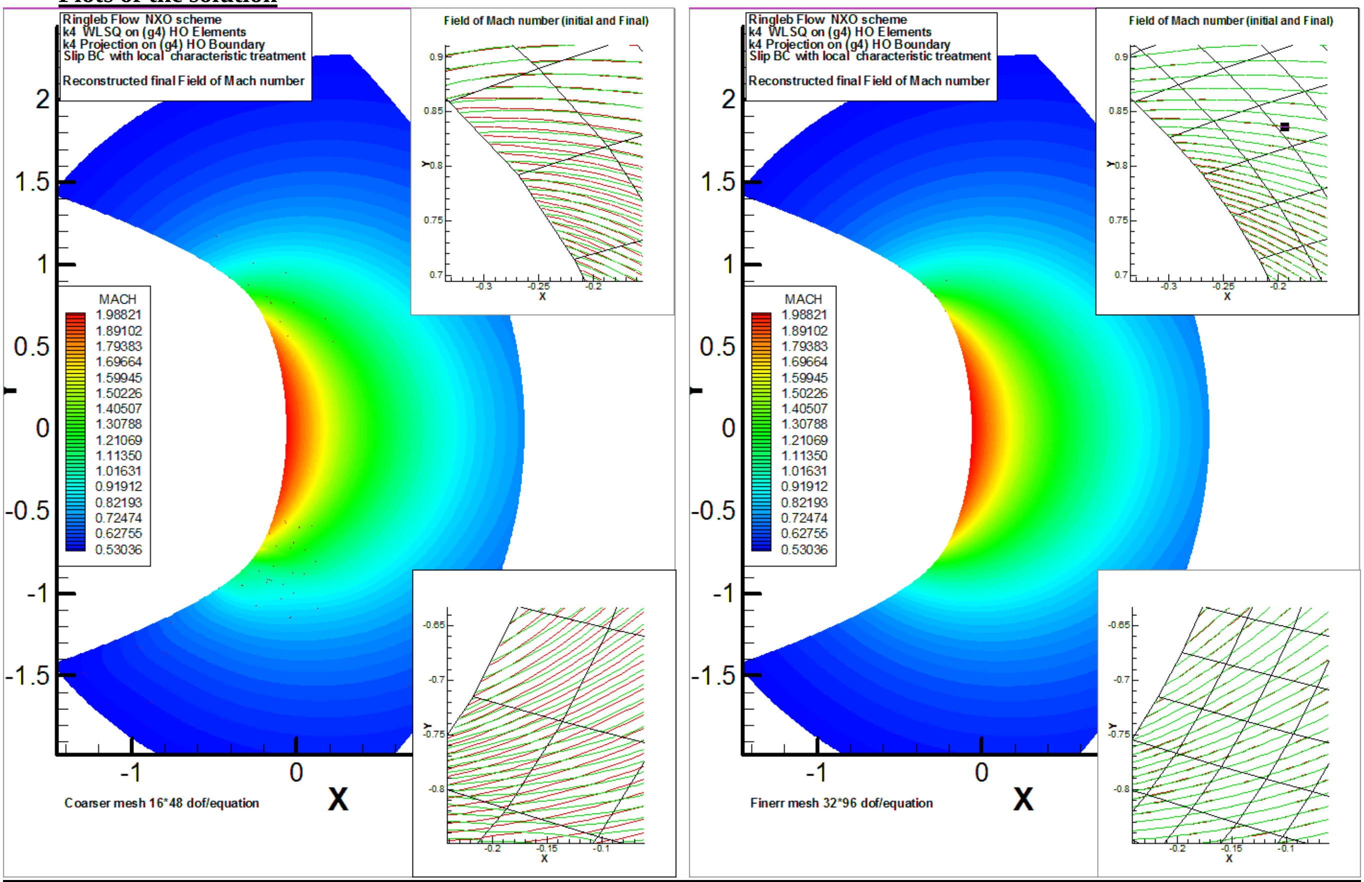
Grid	L2-Error(ro)	L2-Error(roU)	L2-Error(roV)	L2-Error(roEt)	L2-Error(s)
3 (16*48)	1.07e-3	4.83e-4	4.12e-4	1.78e-3	3.06e-4
cvg	***-3.74***	***-3.64***	***-3.82***	***-3.65***	***-3.90***
4 (32*96)	8.02e-5	3.88e-5	2.93e-5	1.42e-4	2.05e-5
cvg	***-3.48***	***-3.00***	***-3.04***	***-3.44***	***-3.53***
5 (64*192)	7.20e-6	4.85e-6	3.57e-6	1.31e-5	1.77e-6

Higher convergence orders are also reached for the k4 formulation. The error levels are only slightly lower than for the k3 formulation.
The convergence order still differs from the theoretical order, since the reconstruction is only done in the least square sense and a weight distribution must be applied to ensure a stable discretization, especially in the stencils near the boundaries.

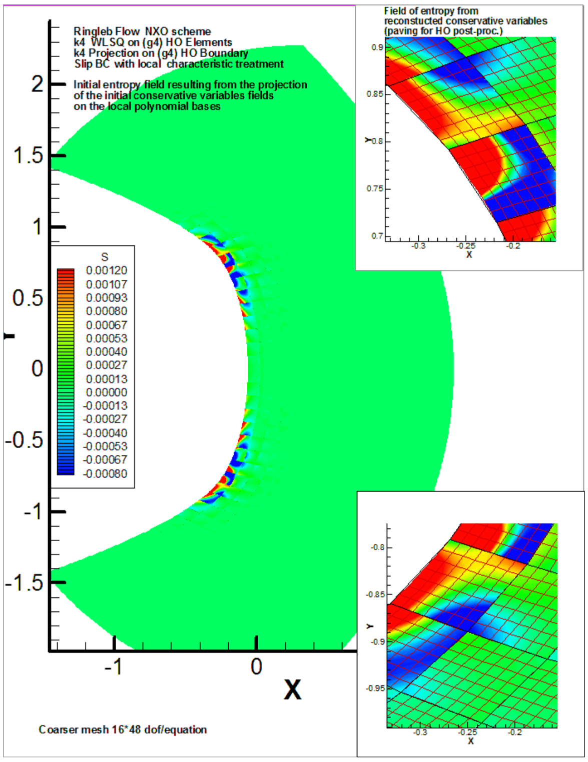
The computing time to evaluate 100 times 250 000 residuals is the following :
k2 : 47.4 taubench Units, k3 : 73.2 taubench Units, k5 : 124.6 taubench Units

<u>Estimated CPU effort to convergence at k5</u> : 68 tbU on grid3, 408 tbU on grid4, 2450 tbU on grid 5.

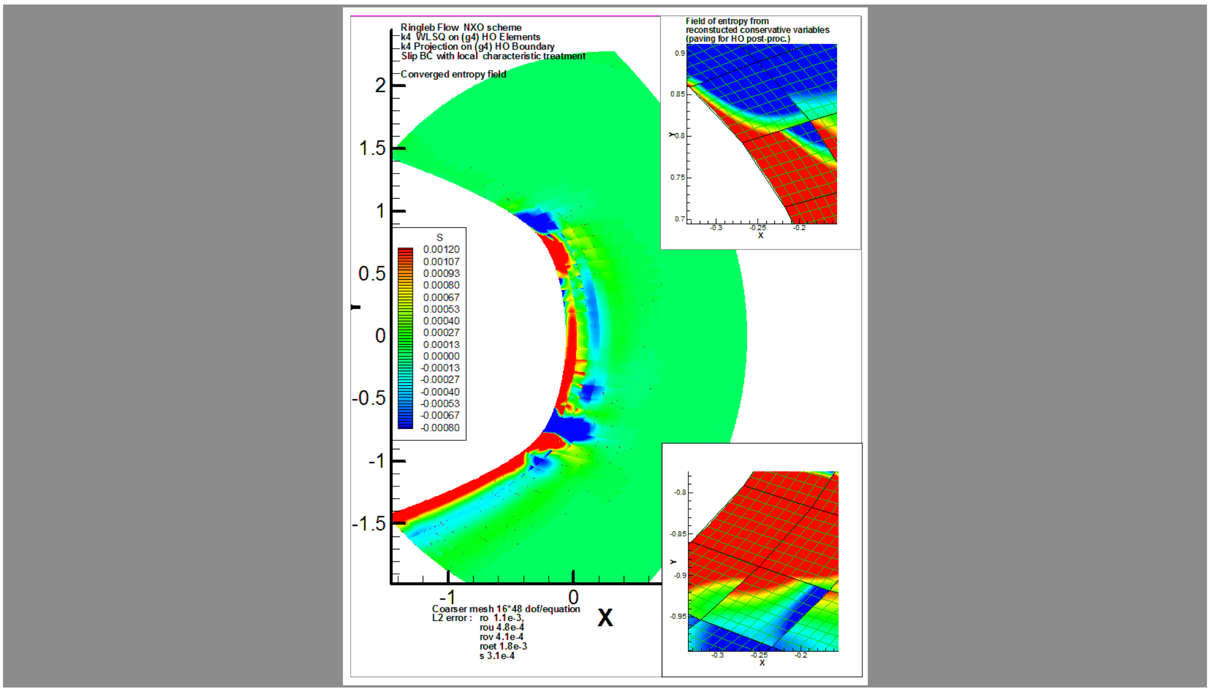
Plots of the solution

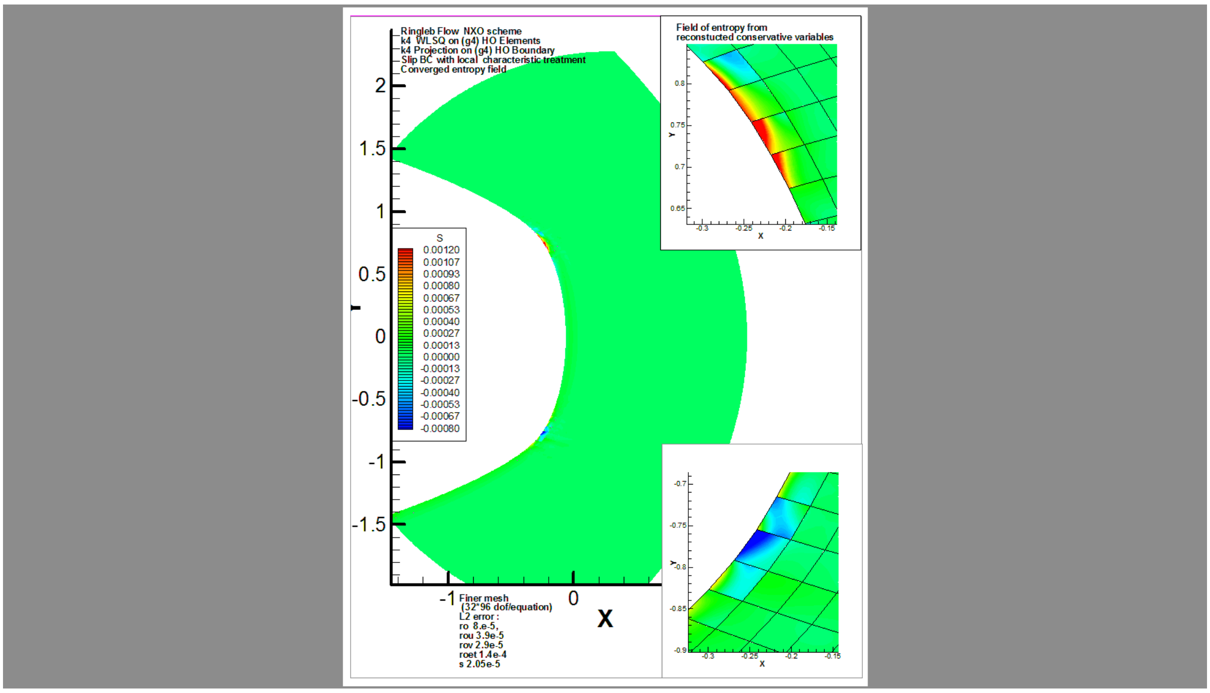


Reconstructed fields of Mach number on grids 3 and 4.
Red isovalues = reconstructed field from the cell-averaged exact solution, green isovalues = reconstruction from the computed fields.

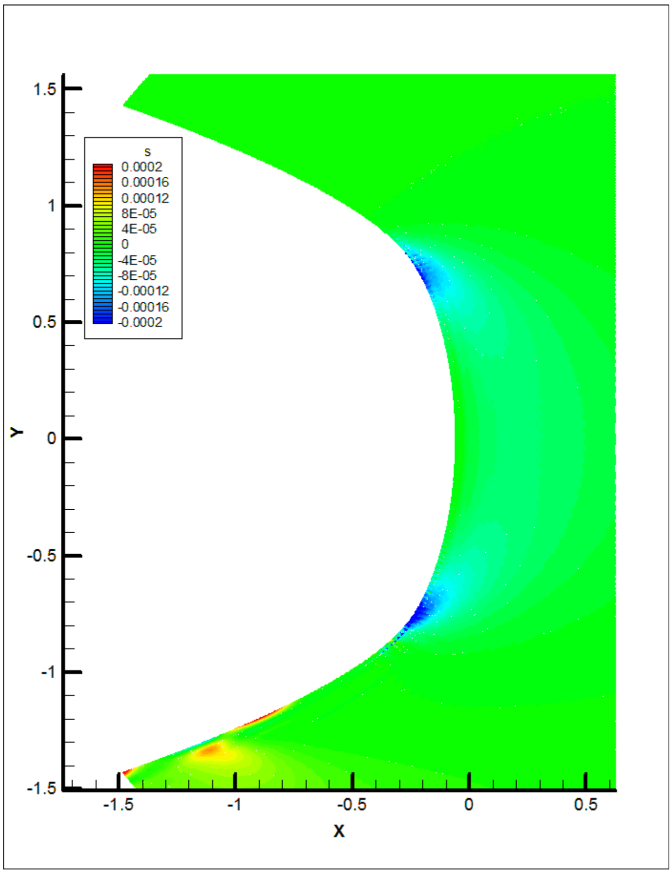


Reconstruction of the entropy field on the coarsest grid from the exact solution.
Exact volume averaged conservative variables fields projected on the polynomial bases.
Representation of the reconstructed field in the central cell of each stencil (discontinuous representation).

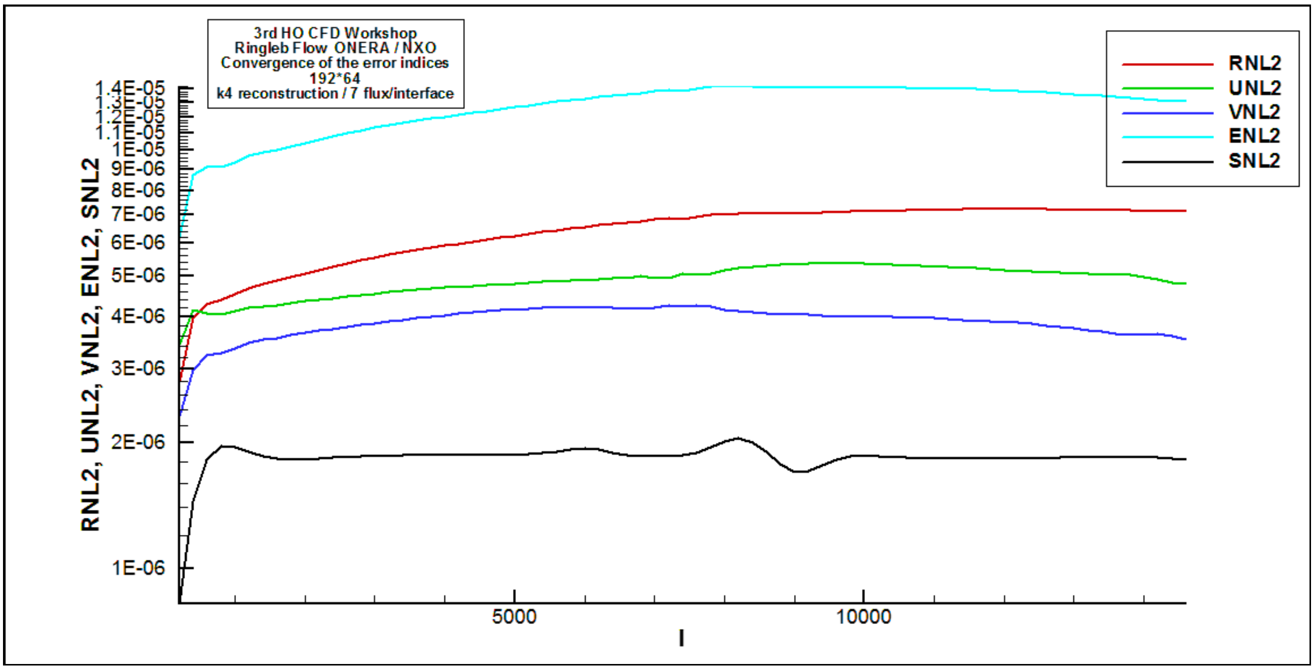




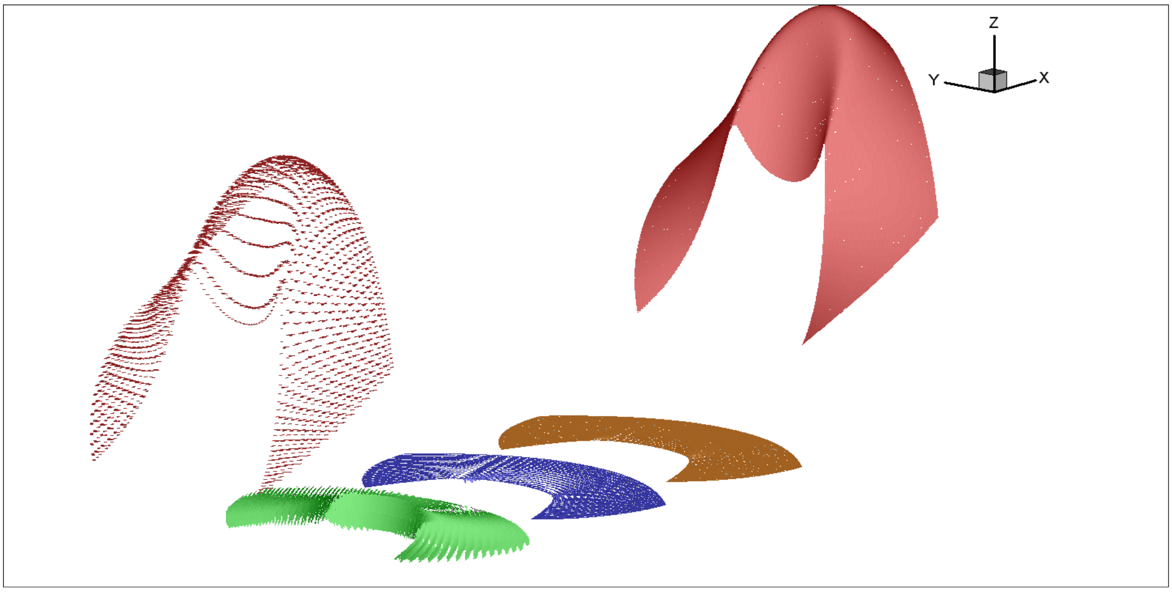
Entropy fields from the converged solution fields on grids 3 and 4 (from the polynomial reconstructions of the conservative variables). Same levels of isovalues.



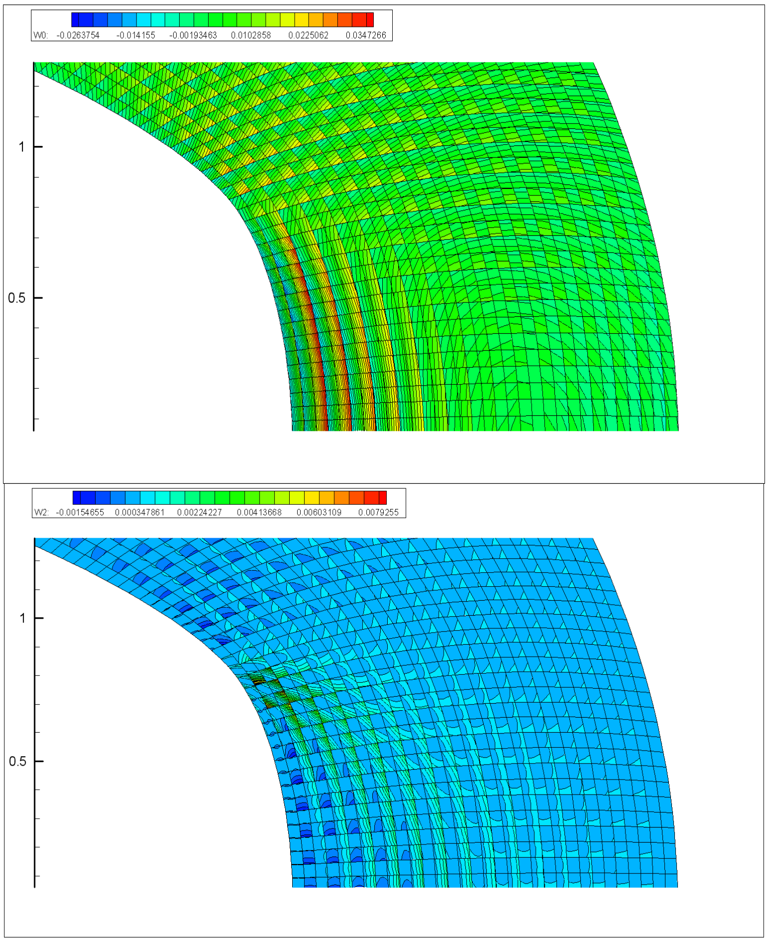
Entropy field from the converged solution fields on grid 5 (from the polynomial reconstructions of the conservative variables).



Convergence of the error indicators in the computation on grid 5, highest accuracy



Spectral content of the reconstructed Y_momentum on grid4 (32*96), k4 reconstruction. Left to right : mode 0 (cell-average), mode1 (linear variation), mode 2, mode 3 and full reconstructed solution in each stencil-central cell.
Discontinuous reconstructed solution across the interfaces (enables the charactericstic-upwind fluxes scheme)



Linear mode on density (top), Quadratic mode on Y-momentum (bottom)

Reference 1 : **J.-M. Le Gouez, V. Couaillier, F. Renac**
High Order Interpolation Methods and Related URANS Schemes on Composite Grids.
48th AIAA Aerospace Sciences Meeting -Orlando, -USA (04-07 Jan 2010), AIAA-2010-513