HYBRID SIMULATION OF BOILING WATER REACTOR DYNAMICS USING A UNIVERSITY RESEARCH REACTOR

NUCLEAR REACTOR SAFETY

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A "hybrid" reactor/simulation (HRS) testing arrangement has been developed and experimentally verified using The Pennsylvania State University (Penn State) TRIGA Reactor. The HRS uses actual plant components to supply key parameters to a digital simulation (and vice versa). To implement the HRS on the Penn State TRIGA reactor, an experimental or secondary control rod drive mechanism is used to introduce reactivity feedback effects that are characteristic of a boiling water reactor (BWR). The simulation portion of the HRS provides a means for introducing reactivity feedback caused by voiding via a reduced order thermal-hydraulic model. With the model bifurcation parameter set to the critical value, the nonlinearity caused by the neutronic-simulated thermal/hydraulic coupling of the hybrid system is evident upon attaining a limit cycle, thereby verifying that these effects are indeed present. The shape and frequency of oscillation (~0.4 Hz) of the limit cycles obtained with the HRS are similar to those observed in operating commercial BWRs. A control or diagnostic system specifically designed to accommodate (or detect) this type of anomaly can be experimentally verified using the research reactor based HRS.

I. INTRODUCTION

The advancement of boiling water reactor (BWR) control and diagnostic system (CDS) technology from the theoretical domain to useful application in existing nuclear power plant operation will require "phased implementation." Prior to a device being adapted for normal plant operation, an intermediate stage is required to bridge the gap between simulation testing and implementation on an actual system. For example, although the general principles of modern control can be demonstrated on a research reactor, such as the Pennsylvania State University TRIGA reactor (PSTR) (Ref. 1), the instrument being tested necessarily has to be designed for use on the TRIGA. Demonstration on a more realistic power reactor would be preferred. Similarly, test signals, the traditional manner in which BWR stability monitors are verified in the United States,[2] cannot incorporate some of the "real world" (e.g., measurement) uncertainties, which could lead to erroneous readings when being used on the actual system. The verification and validation of CDS systems prior to implementation on an operating BWR requires a significant amount of analysis, including experimentation in a more realistic operating environment to appropriately assess their capabilities.

The past three decades have seen a steady decline in the number of experimental nuclear reactors operated in the United States. Facilities such as the Experimental Boiling Water Reactor, which provided the developing nuclear industry with a significant amount of operating experience and engineering insight, laid the foundation for the design and operation of the current generation of BWRs. This trend has been paralleled by an increase in the computational power and sophistication of digital computers, with a corresponding increase in the use of digital simulations to approximate the behavior of the desired process. The ability of these simulations to provide "best estimates" ultimately depends on empirically based correlations, which provide parameters such as two-phase friction multipliers used to estimate plant limiting parameters such as minimum critical power ratio. However, experimentally based bench marking is still required to verify that these programs are providing an accurate description of the process.[3] The U.S. Nuclear Regulatory Commission (NRC) requires vendors to base reactor thermal-hydraulic design code validation on experimentally obtained data. Purdue University's planned simplified boiling water reactor test loop is an example of a test facility to be used to generate this type of experimental data. A similar requirement is foreseen in the area of CDS development, such as BWR stability monitoring equipment. For example, it is desirable for a controller being tested to "see" input signals from actual reactor instrumentation and send calculated output to an actual reactivity control device—allowing the controller to be tested in an environment better approaching a realistic power reactor. In the context of BWR stability monitoring, certain characteristics of the reactor noise signal could be altered to provide a means for determining whether the instrument is capable of detecting a developing limit cycle (i.e., instability) out of a seemingly normal noise signal. No facility exists in the United States where CDS specifically designed for BWRs can be experimentally tested. To accommodate this need, a CDS test bed, which allows the experimental verification of these instruments in a more realistic environment, has been developed. The objective of this paper is to document the background, development, and experimental verification of a hybrid reactor/thermal hydraulic simulation (HRS) CDS testing arrangement.

The HRS (Fig. 1) uses actual plant components to supply key parameters to a simulation (and vice versa). To implement the HRS on the PSTR, an experimental or secondary control rod drive mechanism is used to introduce reactivity feedback effects that are characteristic of an alternate reactor type. For example, reactivity feedback caused by voiding (a BWR characteristic) is not inherent in the design of the TRIGA. Indeed, the nuclear Doppler effect, which reactors designed in the United States depend on for maintaining stability, is not the primary temperature feedback mechanism.[4] The digital simulation portion of the HRS provides a means for introducing this uncharacteristic behavior while negating the natural feedback response of the TRIGA at power levels where the natural temperature feedback would have a noticeable effect. A thermal-hydraulic simulation of a BWR is used to calculate the reactivity feedback effects caused by fuel temperature or voiding. The calculated feedback is converted to the appropriate signals required by a reactivity adjustment mechanism, e.g., a secondary control rod (SCR) situated in the center of the PSTR core. The resulting core power signal is fed back to the simulation to complete the loop. In BWRs, the nonlinear interaction between the thermal-hydraulic aspects of the core and the core neutronics produce a phenomenon known as limit cycles. The HRS, employing a reduced order model[5] to represent the essential thermal-hydraulic characteristics of an operating BWR, has successfully reproduced power fluctuations similar to those exhibited by BWRs experiencing the limit cycle phenomenon.[6] The overall hybrid reactor/

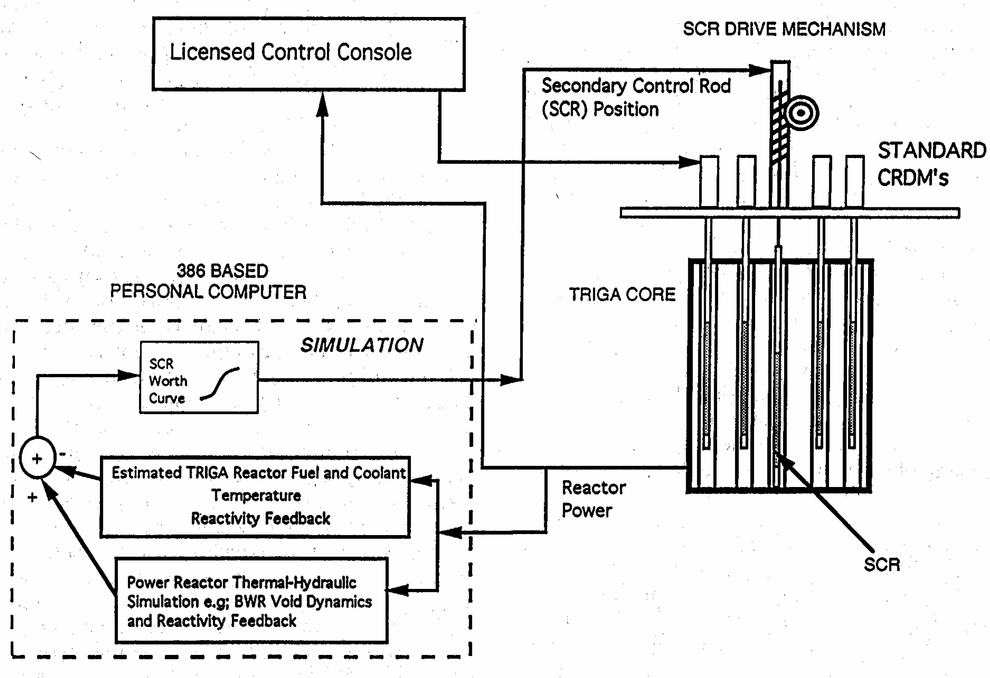

Fig. 1. Hybrid reactor/simulation arrangement.

simulation facility concept is analogous to the North Carolina State University freon pressurized water test loop.[7] In that facility, the reactor kinetics portion of a power plant is simulated with a digital computer, and a physical model of the process is used to obtain the thermal-hydraulic response of a power plant. In the TRIGA/simulation facility, the complementary operation is performed where the thermal-hydraulic response is simulated while the reactor kinetics is generated via the physical system.

II. NONLINEAR BOILING WATER REACTOR DYNAMICS AND THE HYBRID REACTOR SIMULATOR

Like most physical systems, BWRs are inherently nonlinear. Unlike the hypothetical linear system, nonlinear systems can exhibit a variety of additional interesting behaviors. Under certain conditions a system may exhibit stable linear behavior, typical nonlinear characteristics such as limit cycle formation, or even ultrasensitivity to initial conditions known as chaos. Feigenbaum[8] pointed out that for a system to enter this latter regime, an orderly progression of parameters may occur. In seemingly dissimilar nonlinear systems, "universal" constants may be derived, which quantify these changes in system behavior known as bifurcations. In some systems this progression may be barely noticeable with the system seeming to "slip" from linear stability to chaos. In other engineered systems, where the values of key plant parameters (bifurcation parameters) tend to change significantly slower (i.e., BWR), this type of radical change in behavior is extremely unlikely[9] because of the physical limitations of the system. Indeed, if an automatic controller or stability monitor is successful in its designated task, the system should remain within the linear region of operation and never attain the "first step" in the path to chaos, the limit cycle.

In a BWR, nonlinearities may physically manifest themselves in the dependence of system parameters on the system states. For example, in a conduction heat transfer system, the temperature difference between a substance and its surroundings is multiplied by the material's thermal conductivity, which itself is dependent on the temperature of the material, thus producing a nonlinear system. For a given temperature range, assuming that the value of the thermal conductivity is constant usually provides a reasonable estimate of the heat transfer through the material. For this simple thermal system, the temperature dependence on the thermal conductivity would probably have little effect on the dynamic behavior of the heat transfer. The influence of these nonlinearities is more apparent during certain operating conditions that result in relatively large changes in the system's variables or when key parameters change over time. As mentioned earlier, a significant change in the system behavior is observed upon attaining the critical value of a key system parameter (or a combination of system parameters), otherwise known as a bifurcation parameter.

In a nuclear system, the useful and popular reactivity concept permits several effects, which contribute to the nonlinear behavior of the system, to be grouped together. Reactivity provides a means for the feedback effects caused by temperature (i.e., neutron flux) and moderator density to be incorporated into the dynamic calculation of the neutron density. The point kinetics approximation assumes that the neutron flux, which is a function of both space and time, can be separated into a time-dependent amplitude function $n(t)$ and a function describing the spatial distribution $\Psi(r)$. The standard form of the point-kinetics equations for one delayed neutron group are (see Nomenclature on p. 143)

$$\frac{dn(t)}{dt} = \frac{[\rho(t) - \beta]}{\Lambda} n(t) + \lambda c(t) \tag{1}$$

and

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c(t) \ , \tag{2}$$

where the reactivity $\rho(t)$ provides feedback caused by system variables such as fuel temperature and moderator density as well as control system input through control rod positioning. The product of the neutron density and the reactivity provide the system nonlinearity. In a BWR, the feedback caused by moderator density fluctuations (i.e., voiding) results in the limit cycle behavior discussed earlier. It is precisely this effect, as well as the power signal characteristics observed during normal operation, that must be accurately represented to verify CDSs specifically designed for BWRs, such as stability monitors.

III. THE HRS BWR MODEL

With regard to the goal of the HRS, i.e., to enhance the validation process of BWR CDSs, the primary issue is the hybrid system's ability to replicate the crucial dynamic characteristics of the system. In the case of BWR instabilities, the following features must exist, i.e., the system being tested must see the following:

1. the essential physics of the system under observation

2. the same transitory behavior from the stable linear to the nonlinear regime of operation as found in actual systems

3. the shape of the fully developed limit cycle must resemble that found in actual systems including the characteristic pulses observed during large amplitude oscillations

4. the frequency of limit cycle oscillation must fall within the relatively narrow band found during actual instability incidents, 0.3 to 0.6 Hz
5. the simulation must not introduce any dynamic effects not encountered in the actual system, such as the modifications necessary to ensure numerical stability, i.e., numerical damping, as is the case in large thermal-hydraulic simulation codes
6. the system must run in real time.

The frequency response diagram (i.e., transfer function) essentially contains all of the linear and an indication of some of the nonlinear, dynamic characteristics of the system. For example, resonant frequencies shown on the diagram approximately correspond to the characteristic frequency of a limit cycle, if it were to occur. It would seem that if one were primarily interested in the behavior of the system up to the point of limit cycle formation, say for the purposes of developing a simplified model, all of the necessary information would be contained in the frequency response diagram. Thus was the rationale behind the well-publicized reduced order BWR model developed by March-Leuba.[5]

The dynamics of an actual BWR (Vermont Yankee) are empirically obtained via analysis of an average power range monitor signal.[10] This implies, for the purposes of model development, that the reactor may be effectively analyzed as a lumped parameter system. The system dynamics obtained via the transfer function, e.g., power fluctuations, are considered to be integral quantities over the volume of the reactor. With this constraint, a lumped parameter model was derived, which contained the essential physical characteristics necessary for the phenomenon investigated, i.e., BWR oscillations. The oscillatory phenomenon observed in BWRs is caused by the coupling between the thermal hydraulics or coolant density and neutronics of the core. Using the LAPUR frequency domain code[11] and the definition of the closed loop transfer function for a linear system

$$\frac{O(s)}{I(s)} = \frac{G(s)}{1 + G(s)H(s)} , \tag{3}$$

the physical origin of each pole and zero of an empirically determined transfer function was verified via a sensitivity analysis. Thus, the poles associated with the fuel temperature and channel thermal-hydraulic (i.e., void reactivity feedback) response were identified. With the assumed structure of the one delayed group point-kinetics equations for the neutron dynamics, the following set of linear differential equations was used to match the experimentally determined transfer function:

$$\frac{dn_r(t)}{dt} = -\frac{\beta}{\Lambda} n_r(t) + \lambda c_r(t) + \frac{\rho(t)}{\Lambda} , \tag{4}$$

$$\frac{dc_r(t)}{dt} = \frac{\beta}{\Lambda} n_r(t) - \lambda c_r(t) , \tag{5}$$

$$\frac{dT(t)}{dt} = a_1 n_r(t) - a_2 T(t) , \tag{6}$$

$$\frac{d^2\rho_\alpha(t)}{dt^2} + a_3 \frac{d\rho_\alpha(t)}{dt} + a_4\rho_\alpha(t) = KT(t) , \tag{7}$$

$$\rho_{T_f}(t) = DT_f(t) , \tag{8}$$

and

$$\rho(t) = \rho_\alpha(t) + \rho_{T_f}(t) . \tag{9}$$

These equations can be represented by the following block diagram (Fig. 2). The coefficients of Eqs. (4) through (8) are listed in Table I.

If Eq. (4) (i.e., the linear version of the neutron kinetics equation) is reconstituted using Eq. (1) (the nonlinear version with the appropriate normalization; see Nomenclature), a nonlinear system results. As one might expect, the variation of the feedback reactivity caused by voiding ρ_α uses the fuel temperature as its forcing function. The feedback term K in Eq. (7) is analogous to a bifurcation parameter described earlier. Below the critical value, the system appears to behave in a linear fashion. At and above the critical value, the system nonlinearity, essentially the product of $\rho(t)$ and $n(t)$ in Eq. (1), results in the formation of a limit cycle. Further increases in K result in a cascade of period doubling bifurcations.[8] March-Leuba has shown that this nonlinear system exhibits the universal behavior associated with the progression to chaos. Physically, the constant K is related to the core average void and heat transfer coefficients. Equations (6) and (7) provide the BWR feedback characteristics necessary for use in the HRS.

III.A. A Physical Interpretation of the HRS Thermal-Hydraulic Model

Although crafted from the actual observed system behavior, the physical origin of the coefficients and

TABLE I

Coefficient Values Used in Reduced Order BWR Model

Parameter	Value
a_1 (K/s)	25.04
a_2 (s^{-1})	0.23
a_3 (s^{-1})	2.25
a_4 (s^{-2})	6.82
D (K^{-1})	-2.52×10^{-5}
β	0.0056
Λ (s)	4.00×10^{-5}
λ (s^{-1})	0.08

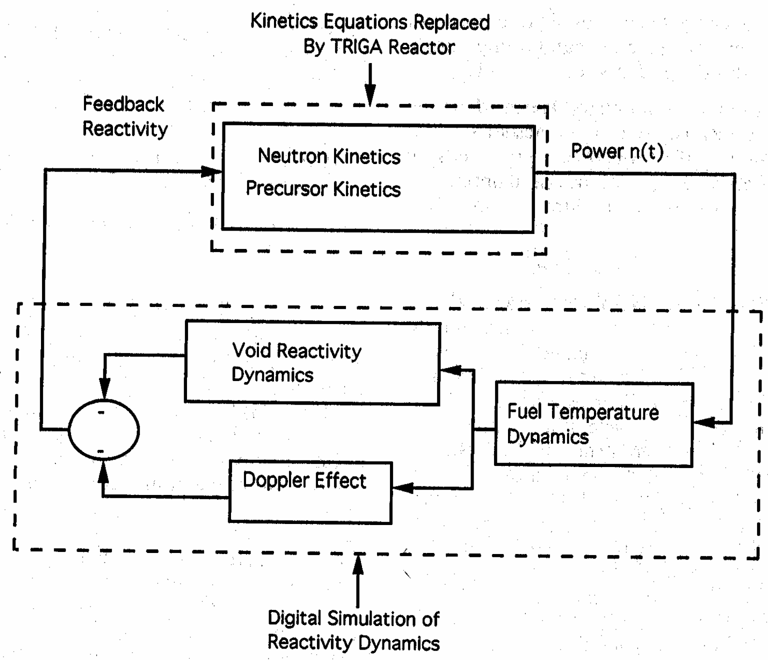

Fig. 2. Block diagram of BWR dynamics.

structure of Eqs. (6) and (7) may be unclear. An understanding of the effect of various parameters (e.g., thermal hydraulic, geometric, and so forth) on the system dynamics, as well as a feel for the origin of the coefficients, lies with a lumped-parameter first principles development of these equations. The following presents the key points of the derivation.[11,12]

Consider the BWR neutron, precursor, fuel temperature, and void dynamics. Assuming that the neutronic behavior throughout the core is separable in time and space, i.e., the point-kinetics approximation for one effective delayed neutron group [Eqs. (1) and (2)], a lumped-parameter development implies that parameters such as the fuel temperature and heat flux are average quantities integrated over their respective regions. For example, the average value for the lumped fuel temperature (i.e., neglecting axial variations in temperature) at a given height in the core would be

$$T(t)_{fuel} = \frac{1}{V_{fuel}} \int_{V_{fuel}} T(r,t)\, dV_{fuel} \ . \tag{10}$$

Development of the fuel temperature dynamics equation, i.e., the forcing function for void production, is performed via an energy balance within the fuel region. The time rate of change of the fuel temperature is proportional to the difference between the heat produced by the fuel and that transferred to the coolant. The first-order equation is thus

$$V_f Q_f''' - 2\pi r_f HU(T_f - T_c) = C_p m_f \frac{dT_f}{dt} \ . \tag{11}$$

Rearranging and linearizing the foregoing equation provides an equation of the same form as Eq. (6):

$$\frac{d\Delta T_f}{dt} = \frac{Q_f'''}{\rho_f C_p} - \frac{2U}{\rho_f C_p r_f} \Delta T_f \ . \tag{12}$$

The linearized void dynamics are developed via the conservation of mass and energy equations for the channel, i.e.,

$$\frac{\partial[(1-\alpha)\rho_l + \alpha\rho_g]}{\partial t} + \frac{\partial G}{\partial z} = 0 \tag{13}$$

and

$$\frac{\partial[(1-\alpha)\rho_l h_l + \alpha\rho_g h_g]}{\partial t} + \frac{\partial[(1-x)h_l G + x h_g G]}{\partial z} = Q \ . \tag{14}$$

Upon perturbing the core average void fraction, mass flux, quality, and heat transferred to the coolant and neglecting terms higher than first order, Eqs. (13) and (14) are combined to obtain

$$\frac{\partial \Delta\alpha(t,z)}{\partial t} + V_0 \frac{\partial \Delta\alpha(t,z)}{\partial z} = \frac{\Delta Q}{H_0} , \tag{15}$$

where

$$V_0 = \frac{G_0(h_g - h_l)\dfrac{dx}{d\alpha}}{(\rho_g h_g - \rho_l h_l) + [h_l(1-x_0) + x_0 h_g](\rho_l - \rho_g)} \tag{16}$$

and

$$H_0 = (\rho_g h_g - \rho_l h_l) + [h_l(1-x_0) + x_0 h_g](\rho_l - \rho_g) . \tag{17}$$

Equation (15) is a hyperbolic distributed parameter (distributed in z) partial differential equation. Thus, V_0 represents the degree at which voids are transferred along the spacial coordinate z, i.e., the void propagation velocity.[13] The value H_0 may be considered to be the pseudo enthalpy of the two-phase mixture. In the context of the present analysis, the distributed nature of $\alpha(t,z)$ (i.e., spatial dependence) will be lumped over the height of the core H. To obtain an ordinary differential equation in z, the Laplace transform of Eq. (15) is performed. An integrating factor is applied to the resulting differential equation to yield

$$\Delta\alpha(z,s) = e^{sz/V_0} \int_0^z e^{-sz'/V_0} \frac{\Delta Q}{H_0} dz' . \tag{18}$$

Consistent with the level of approximation (i.e., point kinetics approximation) the variation in the heat transferred to the coolant is assumed separable in time and space

$$\Delta Q(z,s) = \Phi_0(z)\Delta q(s) . \tag{19}$$

To translate void fraction perturbations into reactivity fluctuations, first-order perturbation theory is employed on the Boltzmann transport equation,[14] producing the resulting general expression for the reactivity

$$\Delta\rho_\alpha = \int_0^H \Phi_0^+(z)\left(\frac{\partial\rho}{\partial\alpha}\right)\Delta\alpha\Phi_0(z)\, dz , \tag{20}$$

where

$$\frac{\partial\rho}{\partial\alpha} = \int dE' \int d\Omega' \times \frac{\partial\left[\Sigma_s(z,t,E':E,\Omega':\Omega) - \dfrac{1}{4\pi}\Sigma_T(E',z,t)\delta(E-E')\right]}{\partial\alpha} \tag{21}$$

and

$$\int_0^H \Phi_0(z)\Phi_0^+(z)\, dz = 1 . \tag{22}$$

The term $\Phi_0^+(z)$ is the solution to the one-dimensional adjoint transport equation.[13] If one-speed diffusion theory is assumed valid, $\Phi_0(z)$ is proportional to $\Phi_0^+(z)$. Combining Eqs. (18), (19), (20), and (22) yields

$$\frac{\Delta\rho_\alpha(s)}{\Delta q(s)} = K \int_0^H \int_0^z e^{-s(z-z')/V_0} dz'\, dz , \tag{23}$$

where the value of the coefficient K, integrated over the height of the core, is

$$K = \left\langle \frac{\Phi_0^3}{V_0} \frac{\partial\rho}{\partial\alpha} \frac{1}{H_0} \right\rangle . \tag{24}$$

Equation (23) is integrated to yield

$$\frac{\Delta\rho_\alpha}{\Delta q} = K\left[\frac{V_0 H}{s} - \frac{V_0^2}{s^2}(1 - e^{-sH/V_0})\right] . \tag{25}$$

For the frequencies of interest, the exponential term can be approximated by a Padé expansion of order 2

$$P_{2,2}(x) = \frac{1 - \dfrac{x}{2} + \dfrac{x^2}{12}}{1 + \dfrac{x}{2} + \dfrac{x^2}{12}} \tag{26}$$

with the perturbations in the heat transferred to the coolant related to the fuel temperature perturbations via

$$\Delta q = U(2\pi r_f H)\Delta T_f . \tag{27}$$

The void reactivity to fuel temperature transfer function is represented by the following:

$$\frac{\Delta\rho_\alpha}{\Delta T_f} = \frac{K1 H^2\left(s + \dfrac{6}{\tau}\right)}{s^2 + \left(\dfrac{6}{\tau}\right)s + \dfrac{12}{\tau^2}} , \tag{28}$$

where τ, the bubble residence time in the core, is defined as the height of the core divided by the propagation velocity V_0 (assuming slip ratio equal to 1.0, i.e., homogenous two-phase flow). Taking the inverse Laplace transform of the previous equation gives

$$\frac{d^2\Delta\rho_\alpha(t)}{dt^2} + \frac{6}{\tau}\frac{d\Delta\rho_\alpha(t)}{dt} + \frac{12}{\tau^2}\Delta\rho_\alpha(t) = K1 H^2\left(\frac{d\Delta T_f}{dt} + \frac{6}{\tau}\Delta T_f\right) , \tag{29}$$

where

$$K1 = \left\langle \frac{\Phi_0^3 \dfrac{\partial\rho}{\partial\alpha} U 2\pi r_f}{H_0} \right\rangle . \tag{30}$$

The coefficients of Eq. (29) [which is similar in form to Eq. (7), the core-averaged void dynamics] shed some light on the behavior of the physical system. Specifically the bubble residence time τ has a direct effect on the system damping, i.e., the coefficient of the dissipative term [i.e., the first derivative of $\rho_\alpha(t)$] in Eq. (29). As the core flow decreases, τ increases, reducing the system damping and, consequently, stability. This is the case in operating BWRs prior to a stability incident. The coefficient of the forcing function for Eq. (29) is a function of the heat transfer coefficient that as the amount of boiling increases because of decreased flow (increased τ), increases significantly. Thus, the effect of decreasing the core flow is twofold, i.e., reducing the system damping while simultaneously increasing the system forcing (with the net effect being decreased system stability). The core geometry also has an effect on the system stability through the H^2 term on the right side of Eq. (29). An increase in core height increases the amplification of the equation's forcing term, which also aids in decreasing the system damping through the bubble residence time τ.

The development just presented provides a qualitative description of the effects of core coolant flow and geometry on the dynamics of a "point" BWR, i.e., the coefficients a_3, a_4, and K of Eq. (7). The coefficients a_3 and a_4, which are interpreted as being functions of the thermal-hydraulic channel damping ratio and natural frequency, are also dependent on fuel loading configuration and control rod positions.[15] Variations in these coefficients, corresponding to different reactor design or operating conditions, would be observed through the system damped natural frequency, i.e., the peak in the noise power spectrum occurring between 0.1 and 1.0 Hz. Good agreement has been found between the third coefficient of Eq. (29) (solely a function of the natural frequency) and experimentally obtained data through correspondence of the calculated bubble residence time (i.e., equating a_4 to the $12/\tau^2$ term) with measured values.[11] The second coefficient, being a function of the damping ratio and the natural frequency, is more dependent on fuel loading and control rod position (essentially three-dimensional effects). This results in a significant difference between the empirically obtained value (e.g., a_3 as presented in Table I) and that calculated using the expression $6/\tau$. However, the term $6/\tau$ does provide an accurate qualitative measure of the influence of τ on the stability of the system. The experimentally obtained values of a_3 and a_4 would, of course, implicitly take these additional factors into account.

IV. HYBRID REACTOR/BWR SIMULATION (HRS) DEVELOPMENT

To demonstrate the HRS (Fig. 1), the feedback dynamics associated with the fuel temperature and the channel thermal hydraulics is simulated via a reduced order model [Eqs. (6) through (9)]. The resulting reactivity caused by voiding (boiling) is converted to an experimental (or secondary) control rod position. The power fluctuations produced by the control rod movement are used by the simulation to drive the simulated temperature (and subsequent boiling) response. Because of factors such as detector field of view,[16] measurement uncertainty, and so forth, this same power fluctuation will result in an attenuated power response on the TRIGA operating console (as well as by the potential controller to be tested). With the bifurcation parameter [K in Eq. (7)] set to the critical value, the nonlinearity caused by the neutronic-simulated thermal-hydraulic coupling of the hybrid system is evident upon attaining a limit cycle, thereby verifying that these effects are indeed present. During prelimit cycle operation, the TRIGA reactor power signal exhibits some additional noise caused by the simulated boiling effects.

The PSTR is a Mark III TRIGA reactor.[a] A light water-cooled and -reflected pool-type research reactor, the TRIGA operates at a maximum constant power of 1 MW and is routinely pulsed to 1000 MW. The hexagonally shaped core of the TRIGA contains 85 fuel elements. The fuel elements contain the zirconium hydride moderator homogeneously combined with the partially enriched uranium fuel. This type of fuel has a very large and prompt negative temperature coefficient because of hardening (shifting from thermal to higher energies) of the neutron spectrum with increases in temperature. This characteristic provides an additional measure of safety when performing experiments and is essentially the mechanism that allows the reactor to be pulsed. Figure 3 shows the PSTR core configuration and control rod locations.

The TRIGA technical specifications allow for a "movable experiment" to be conducted independently of the licensed control and monitoring system (LCMS). The LCMS positions four control rods designated as the regulating rod, safety rod, shim rod, and transient rod, which is used for pulsing. A movable experiment was created by adding a fifth control rod in the central thimble of the reactor. This movable experiment can be manipulated by any means as long as the technical specifications on reactivity insertion rates and maximum reactivity are met. The SCR is used to provide the desired HRS reactivity feedback characteristics for the system under consideration, in this case a BWR.

The simulation, running on a 386 PC, uses the normalized reactor power signal to drive the excess average fuel temperature differential equation [Eq. (6)]. In a BWR, fluctuations in fuel temperature result in core void fraction changes. The combined Doppler and void reactivity is fed back to the TRIGA via the SCR. The temperature and void dynamic equations are numerically integrated via the fourth-order Runge-Kutta algorithm, with a fixed time step of 3 ms. To determine the appropriate SCR speed demand signal, based on the

[a]Of General Atomics.

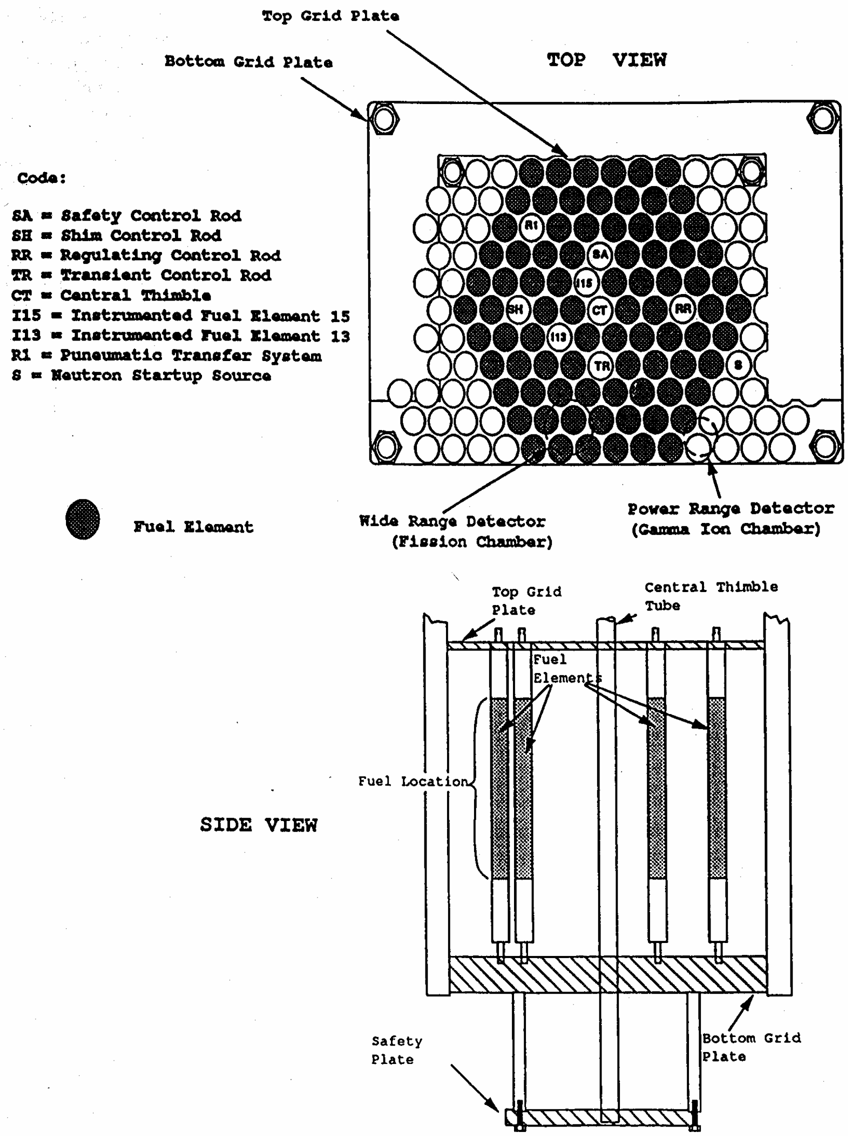

Fig. 3. The PSTR core configuration.

rate of change of reactivity insertion corresponding to the boiling dynamics, the following control rod speed relationship was employed

$$\frac{dZ_{scr}}{dt} = \frac{d\rho}{dt} \cdot \frac{1}{\frac{d\rho}{dz}} , \qquad (31)$$

where $d\rho/dt$ is obtained via differentiation of Eq. (9) (with appropriate substitutions), and $d\rho/dz$ is obtained from the differential SCR worth curve (for the initial HRS demonstration assumed constant over the anticipated SCR travel). The maximum SCR worth is 0.35 \$ with the maximum speed clamped at 30% core length per second (~0.15 \$/s).

A DataTranslation analog to digital (and digital to analog) converter board provides the communication link between the 386 personal computers (i.e., the simulation) and the TRIGA. A potentiometer was used to provide continuous fine tuning of the value of the bifurcation parameter K in Eq. (7). The power signals provided by a fission chamber (shown in Fig. 3) are electronically converted into a logarithmic signal, which is appropriately conditioned for use in the simulation. The required SCR velocity is calculated by the simulation and converted to an analogue voltage. The BWR simulated dynamics are encoded into a C language program.

IV.A. Demonstration of the HRS

The HRS experiment consists of bringing the TRIGA up to a steady-state power level of 500 W with the licensed control console. To eliminate the need for additional SCR reactivity worth to "override" the TRIGA's temperature reactivity feedback effects, the HRS is operated at power levels where these effects would be negligible (i.e., <1000 W). After positioning the SCR in the vertical center of the TRIGA core (location of maximum differential reactivity worth), the licensed console is placed into the manual mode of operation. The potentiometer voltage (i.e., the bifurcation parameter K) is set to a value well below the critical value and the simulation started. Figure 4 shows the filtered TRIGA reactor power derivative response during the transition from normal operation, i.e., the TRIGA with no simulated BWR characteristics, to operation as an HRS. Although it is undetectable when the power signal is being viewed, a marked change is seen in the power derivative (i.e., rate of change of power) trace. As the value of K is increased, a significant increase in SCR movement is observed, i.e., the simulation is responding to the power signal noise and positioning the SCR accordingly. At the critical value of K, a relatively constant amplitude oscillation forms and is observed via the power signal, i.e., a limit cycle (Fig. 5). The observed behavior in the HRS reproduces that observed in stability tests and events.[17,18] The HRS is limited, however, to relatively small amplitude limit cycles because of the low worth of the SCR, which inhibits the "pulse-like" behavior observed during some of the larger amplitude limit cycle events.[19]

Reactor stability analysis typically necessitates the use of a frequency domain approach, i.e., Fourier transformation of the reactor noise signal, to determine the characteristic frequency of a limit cycle oscillation (if one were to exist). As the value of the bifurcation parameter K is increased, corresponding to key core parameters changing over time, the system tends toward instability. This behavior, although present in the

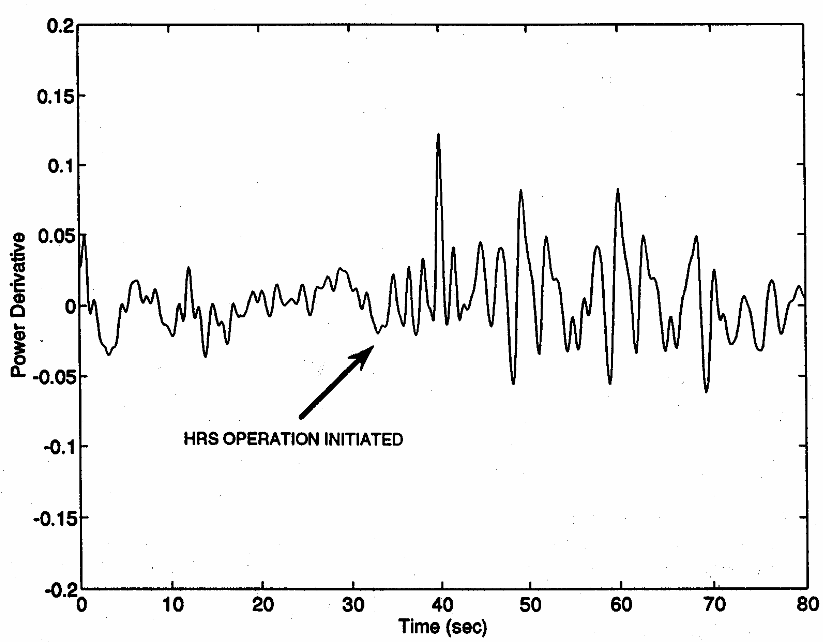

Fig. 4. Filtered rate of change of the power signal from TRIGA during HRS startup (cutoff frequency of 1 Hz).

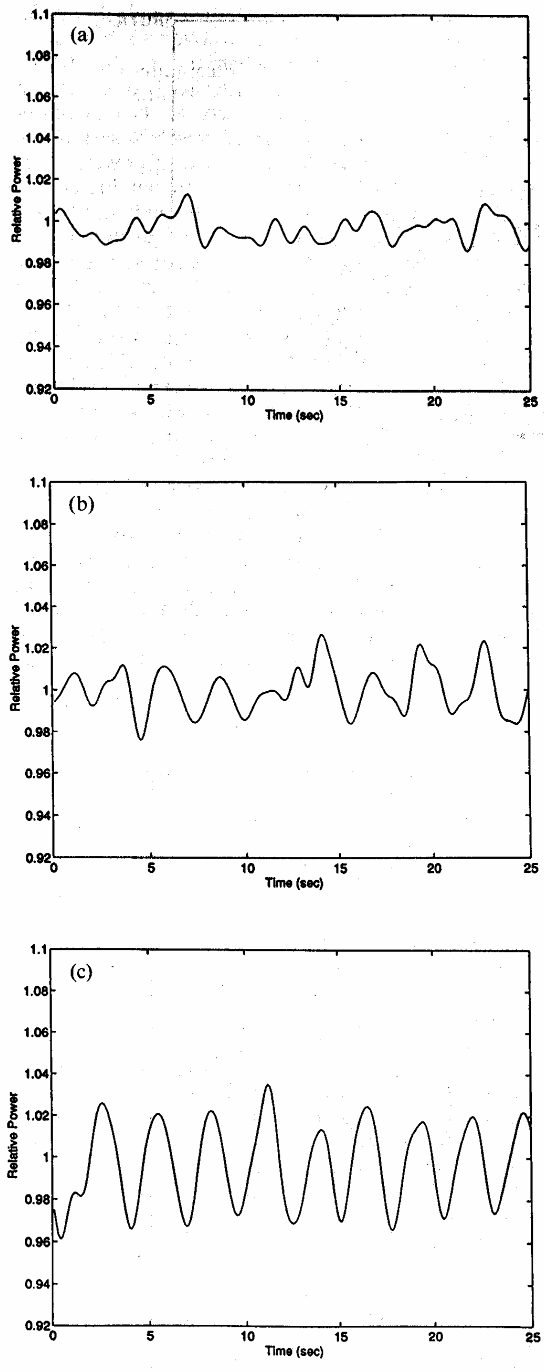

Fig. 5. Evolution to an HRS limit cycle in the time domain, where (a) $K = 0.72K_{crit}$, (b) $K = 0.83K_{crit}$, and (c) $K = K_{crit}$.

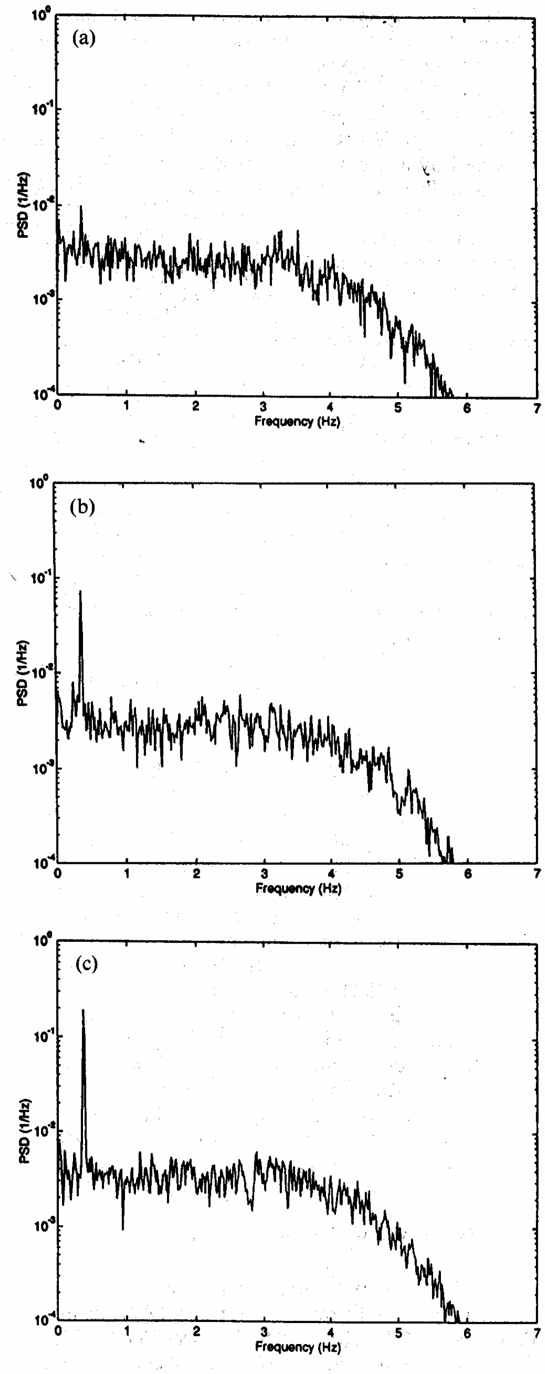

Fig. 6. Evolution to an HRS limit cycle in the frequency domain, where (a) $K = 0.72K_{crit}$, (b) $K = 0.83K_{crit}$, and (c) $K = K_{crit}$.

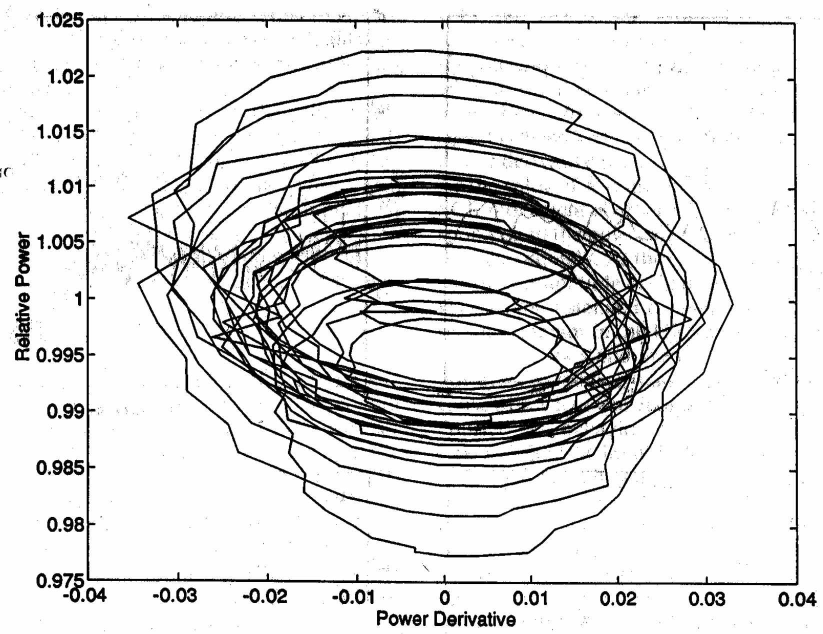

Fig. 7. Fully developed HRS limit cycle in the phase space.

power signal, may be undetectable through observation in the time domain. Thus, there is the need for frequency domain analysis. Figure 6 presents the power spectral density (PSD) of the reactor noise signal during several stages in the evolution of an HRS limit cycle.

As the value of K is gradually increased from its initial value, the power spectrum of the reactor noise displays an increase in the frequency band of interest (0.3 to 0.6 Hz). This is caused by the frequency response of the model (i.e., the simulated boiling reactivity response caused by the noisy power input). As is shown in Fig. 6, a pronounced peak in the power spectrum appears at ~0.4 Hz, within the frequency band typically observed during limit cycle oscillations in operating BWRs. Figure 7 presents a fully developed HRS limit cycle in a portion of the phase space consisting of the relative power and its time derivative.

V. SUMMARY AND CONCLUSIONS

By the addition of appropriate equations and parameters, the nonlinear behavior of a BWR using the PSTR as part of a HRS system has been demonstrated. Specifically limit cycles, a nonlinear phenomenon that has been known to occur in an operating BWR, can be obtained by manipulating a system bifurcation parameter. A bifurcation parameter controls the extent to which simulated voiding and the associated feedback reactivity occur within a reduced order model. A physical interpretation of the model used in the HRS is presented to provide a means of linking several of the key system variables, such as void residence time and core height, to the stability of a BWR. Through a qualitative analysis of this physical interpretation, increasing the void residence time (i.e., decreasing core flow) is seen to decrease the system damping. Increasing the core height is also seen to have a destabilizing effect through the void residence time and the forcing function for the void dynamics.

The reduction of the experimental data in the frequency domain, a technique widely used in BWR operation to monitor stability, was the most appropriate method to validate the HRS in the presence of noisy measurements. The shape and frequency of oscillation of the limit cycles obtained with the HRS are similar to those observed in operating commercial BWRs. During the evolution of an HRS limit cycle, a peak is observed to appear at ~0.4 Hz, thereby verifying that these additional nonlinear effects are indeed present.

It can be concluded that the HRS, in addition to introducing the desired power fluctuations, could be a useful tool in BWR control system development and stability analysis research, i.e., using known noise spectra modification to test the response of BWR instrumentation.

NOMENCLATURE

$a_1 \ldots a_4$ = empirically determined reduced order BWR model coefficients

C_0 = steady-state precursor concentration

c = precursor concentration

c_r = relative precursor concentration [i.e., $(c - C_0)/N_0$]

G = mass flux

G_0 = steady-state mass flux

$G(s)$ = forward loop transfer function

$H(s)$ = feedback loop transfer function

$h_{l(g)}$ = enthalpy of liquid (or vapor) phase

$I(s)$ = Laplace transform of system input

N_0 = steady-state neutron concentration

n = neutron concentration

n_r = relative neutron concentration [i.e., $(n - N_0)/N_0$]

$O(s)$ = Laplace transform of system output

Q = coolant heat absorption rate

Q_f''' = volumetric heat produced in fuel

T = fuel temperature

t = time

U = fuel heat transfer coefficient

x = quality

x_0 = steady-state quality

Z_{scr} = secondary control rod vertical position

Greek

α = void fraction

β = delayed neutron fraction

Δ = state variable deviation from equilibrium

Λ = prompt neutron lifetime

λ = delayed neutron precursor decay constant

ρ = reactivity

ρ_α = feedback reactivity caused by voiding

$\rho_{l(g)}$ = density of liquid (or vapor) phase

ρ_t = feedback reactivity caused by temperature

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