Model-Based Aircraft Engine Performance Estimation

Donald L. Simon
NASA Glenn Research Center

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Outline

- Background – On-board self-tuning engine models
- Problem formulation – optimal selection of model tuning parameters
- Optimal Tuner Selection Methodology: Previous approach and recent enhancements
- Turbofan engine simulation example
- Conclusions

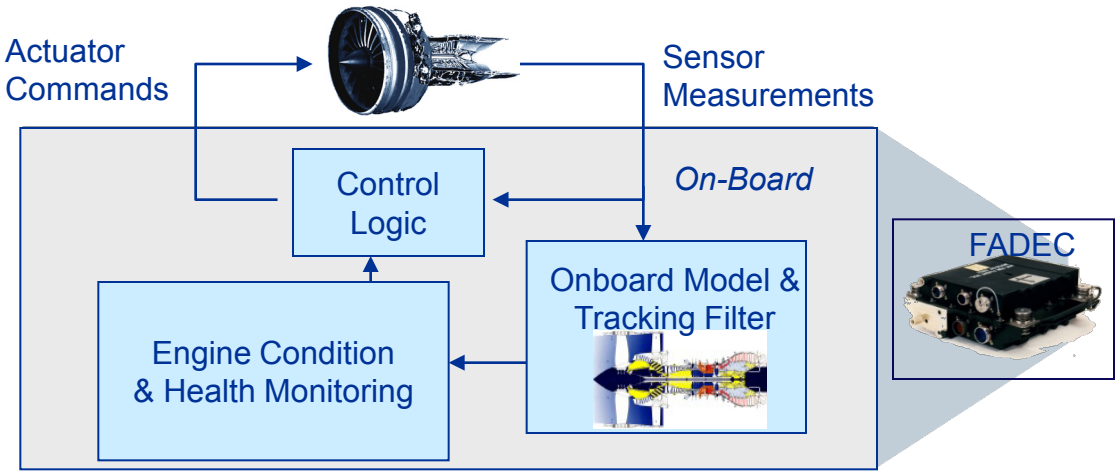
Background

On-board Self-Tuning Engine Models

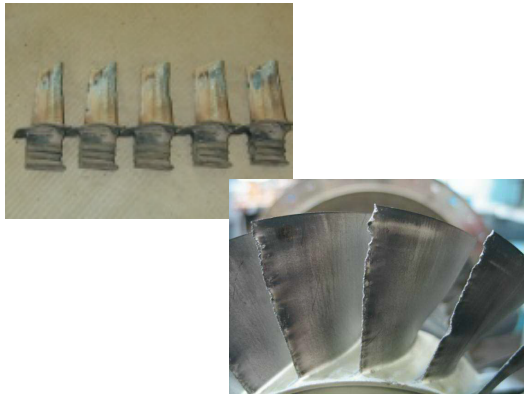
Background:
- Adaptive engine model embedded within on-board real-time computer
- A tracking filter tunes model to match the physical engine performance

Benefits:
- Continuous, real-time engine condition monitoring
- Estimation of unmeasured engine parameters

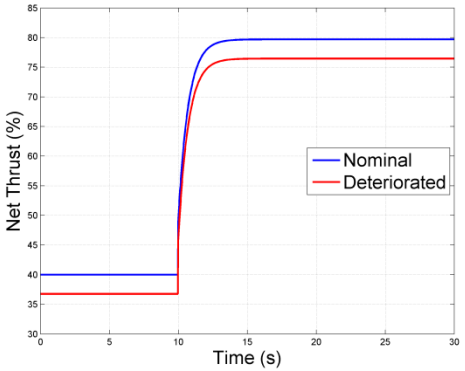
Challenges:
- Underdetermined estimation problem – more unknowns than available sensor measurements



Model-based Diagnostics Architecture



Deteriorated turbomachinery



Deteriorated engine performance

Problem Formulation

State Space Equations

Full-order state space equations

$$\begin{bmatrix} x_{k+1} \\ h_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & L \\ 0 & I \end{bmatrix}}_{A_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_{xh}} u_k + \underbrace{\begin{bmatrix} w_k \\ w_{h,k} \end{bmatrix}}_{w_{xh,k}}$$

$$= A_{xh} x_{xh,k} + B_{xh} u_k + w_{xh,k}$$

State variables

$$y_k = \underbrace{\begin{bmatrix} C & M \end{bmatrix}}_{C_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + D u_k + v_k$$

$$= C_{xh} x_{xh,k} + D u_k + v_k$$

Measured (sensed) outputs

$$z_k = \underbrace{\begin{bmatrix} F & N \end{bmatrix}}_{F_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + G u_k$$

$$= F_{xh} x_{xh,k} + G u_k$$

Unmeasured (auxiliary) outputs

Problem Formulation

State Space Equations

Full-order state space equations

$$\begin{bmatrix} x_{k+1} \\ h_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & L \\ 0 & I \end{bmatrix}}_{A_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_{xh}} u_k + \underbrace{\begin{bmatrix} w_k \\ w_{h,k} \end{bmatrix}}_{w_{xh,k}}$$

$$= A_{xh} x_{xh,k} + B_{xh} u_k + w_{xh,k}$$

$$y_k = \underbrace{\begin{bmatrix} C & M \end{bmatrix}}_{C_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + D u_k + v_k$$

$$= C_{xh} x_{xh,k} + D u_k + v_k$$

$$z_k = \underbrace{\begin{bmatrix} F & N \end{bmatrix}}_{F_{xh}} \underbrace{\begin{bmatrix} x_k \\ h_k \end{bmatrix}}_{x_{xh,k}} + G u_k$$

$$= F_{xh} x_{xh,k} + G u_k$$

Reduced-order state space equations

$$\begin{bmatrix} x_{k+1} \\ q_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & L^* \\ 0 & I \end{bmatrix}}_{Axq} \underbrace{\begin{bmatrix} x_k \\ q_k \end{bmatrix}}_{x_{xq,k}} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_{xq}} u_k + \underbrace{\begin{bmatrix} w_k \\ w_{q,k} \end{bmatrix}}_{w_{xq,k}}$$

$$= A_{xq} x_{xq,k} + B_{xq} u_k + w_{xq,k}$$

$$y_k = \underbrace{\begin{bmatrix} C & M^* \end{bmatrix}}_{Cxq} \underbrace{\begin{bmatrix} x_k \\ q_k \end{bmatrix}}_{x_{xq,k}} + D u_k + v_k$$

$$= C_{xq} x_{xq,k} + D u_k + v_k$$

$$z_k = \underbrace{\begin{bmatrix} F & N^* \end{bmatrix}}_{Fxq} \underbrace{\begin{bmatrix} x_k \\ q_k \end{bmatrix}}_{x_{xq,k}} + G u_k$$

$$= F_{xq} x_{xq,k} + G u_k$$

Kalman filter is formulated based on reduced-order state space equations where health parameter vector, *h*, representing engine performance deterioration is replaced by reduced-order tuner vector, *q*.

Problem Formulation

Underdetermined Estimation: Tuner Selection Approaches

1. Define and apply a reduced-order tuner vector, q, that is a subset of the health parameter vector (conventional approach).
   Here, the dimension of q must be $\leq$ number of sensors.

$$q \subseteq h$$

2. Define and apply a reduced-order tuner vector, q, that is a linear combination of all health parameters*

$$q = V^{*} h$$

Where V^{*} is a linear transformation matrix of dimension $\leq$ number of sensors $\times$ number of health parameters.

Problem Formulation

Kalman Filter Equations

This study applies steady-state Kalman filtering

Required parameters:

System matrices, *A,B,C,D,F,G,L,M,N*
Measurement noise covariance, R
Process noise covariance, Q_{xh}
Transformation matrix, V^*

Reduced-order state process noise covariance

$$Q_{xq} = \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix} Q_{xh} \begin{bmatrix} I & 0 \\ 0 & V^* \end{bmatrix}^T$$

Riccati equation

$$P_\infty = A_{xq} P_\infty A_{xq}^T - A_{xq} P_\infty C_{xq}^T (C_{xq} P_\infty C_{xq}^T + R)^{-1} C_{xq} P_\infty A_{xq}^T + Q_{xq}$$

Steady-state Kalman gain

$$K_\infty = P_\infty C_{xq}^T (C_{xq} P_\infty C_{xq}^T + R)^{-1}$$

a priori state estimate

$$\hat{x}_{xq,k}^- = A_{xq} \hat{x}_{xq,k-1}^+ + B_{xq} u_{k-1}$$

a posteriori state estimate

$$\hat{x}_{xq,k}^+ = \hat{x}_{xq,k}^- + K_\infty \left(y_k - C_{xq} \hat{x}_{xq,k}^- - D u_k \right)$$

Estimate health or auxiliary parameters

$$\hat{x}_{xh,k} = \begin{bmatrix} I & 0 \\ 0 & V^{*\dagger} \end{bmatrix} \hat{x}_{xq,k}^+ \qquad \hat{z}_k = \begin{bmatrix} F & NV^{*\dagger} \end{bmatrix} \hat{x}_{xq,k}^+ + G u_k$$

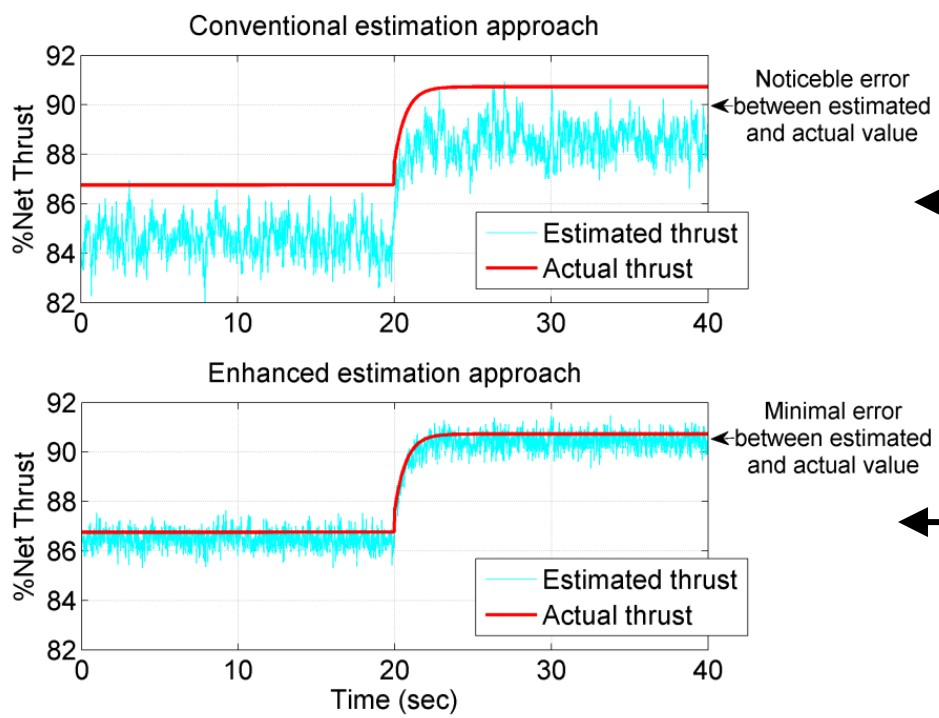
Problem Formulation

Optimal Tuner Selection Methodology: Previous Approach

Previous approach: Performed optimal tuner selection to minimize Kalman filter theoretical mean squared estimation error under steady-state open-loop operation conditions at a single design point.

$q = V^{*}h$ ➡ ***Objective: choose V^{*}, and thereby q, to minimize the Kalman filter estimation error in the parameters of interest***

Thrust estimation accuracy comparison of conventional vs. enhanced estimation approaches



$q \subseteq h$

$q = V^{*}h$

Problem Formulation

Optimal Tuner Selection Methodology: Recent Enhancements

Recent enhancements: The Optimal Tuner Selection Methodology has been updated to incorporate the following enhancements:

1) Minimization of the theoretical Kalman filter mean squared estimation error under steady-state closed-loop operating conditions
2) Selection of "globally optimal" model tuner parameters to minimize the mean squared estimation error over multiple user-specified operating points

1) Minimization of the Kalman Filter Mean Sum of Squared Estimation Errors Under Closed-Loop Operating Conditions

Required information for derivation of error:

- Process, measurement, and health parameter covariance (Q , R, and P_h)
- Engine linear state space equations
- V^* matrix transforming h to q
- Controller linear state space equations

Kalman filter mean sum of squared estimation errors (SSEE) consists of:

Mean squared estimation error bias:

$$\bar{\hat{x}}^{+}_{xq,ss} = \underbrace{\left[\left(I-K_\infty C_{xq}\right)\left(I-A_{xq}\left(I-K_\infty C_{xq}\right)\right)^{-1}\left[A_{xq}K_\infty \quad B_{xq}-A_{xq}K_\infty D\right]+\left[K_\infty \quad -K_\infty D\right]\right]\begin{bmatrix}\left(\bar{C}\left(I-\bar{A}\right)^{-1}\bar{L}+M\right)\\ \left[0 \quad C_c\right]\left(I-\bar{A}\right)^{-1}\bar{L}\end{bmatrix}}_{G_{xq}} h$$

$$\bar{\hat{x}}^{+}_{xq,ss} = G_{xq}h$$

$$\bar{\hat{z}}_{ss} = E\left[\hat{z}_k - z_k\right] = \bar{\hat{z}}_{ss} - z_{ss}$$

$$= \underbrace{\left[\left[F \quad NV^{*\dagger}\right]G_{xq} - \left[\bar{F}\left(I-\bar{A}\right)^{-1}\bar{L}+N\right]\right]}_{G_z} h$$

$$\bar{\hat{x}}^2_{xh,fleet} \equiv E\left[\bar{\hat{x}}^T_{xh,ss}\bar{\hat{x}}_{xh,ss}\right] = E\left[tr\left\{\bar{\hat{x}}_{xh,ss}\bar{\hat{x}}^T_{xh,ss}\right\}\right] = tr\left\{G_{xh}P_hG^T_{xh}\right\}$$

$$\bar{\hat{z}}^2_{fleet} \equiv E\left[\bar{\hat{z}}^T_{ss}\bar{\hat{z}}_{ss}\right] = E\left[tr\left\{\bar{\hat{z}}_{ss}\bar{\hat{z}}^T_{ss}\right\}\right] = trace\left\{G_zP_hG^T_z\right\}$$

Estimation variance:

$$P_{\hat{xq},k} = E\left[\underbrace{\left(\hat{x}^{+}_{xq,k} - E\left[\hat{x}^{+}_{xq,k}\right]\right)}_{\varepsilon_{xq,k}}\underbrace{\left(\hat{x}^{+}_{xq,k} - E\left[\hat{x}^{+}_{xq,k}\right]\right)^T}_{\varepsilon^T_{xq,k}}\right]$$

$$\varepsilon_{xq,k} = \hat{x}^{+}_{xq,k} - E\left[\hat{x}^{+}_{xq,k}\right]$$

$$\varepsilon_{xq,k} = \left(I-K_\infty C_{xq}\right)A_{xq}\left(\hat{x}^{+}_{xq,k-1} - \bar{\hat{x}}^{+}_{xq,ss}\right) + K_\infty\left(y_k - y_{ss}\right)$$

$$P_{\hat{xq},k} = \left[A_{xq} - K_\infty C_{xq}A_{xq}\right]P_{\hat{xq},k}\left[A_{xq} - K_\infty C_{xq}A_{xq}\right]^T + K_\infty RK^T_\infty$$

$$P_{\hat{xh},k} = \begin{bmatrix} I & 0 \\ 0 & V^{*\dagger}\end{bmatrix}P_{\hat{xq},k}\begin{bmatrix} I & 0 \\ 0 & V^{*\dagger}\end{bmatrix}^T \qquad P_{\hat{z},k} = \left[F \quad NV^{*\dagger}\right]P_{\hat{xq},k}\left[F \quad NV^{*\dagger}\right]^T$$

Sum to produce mean (weighted) sum of squared estimation errors

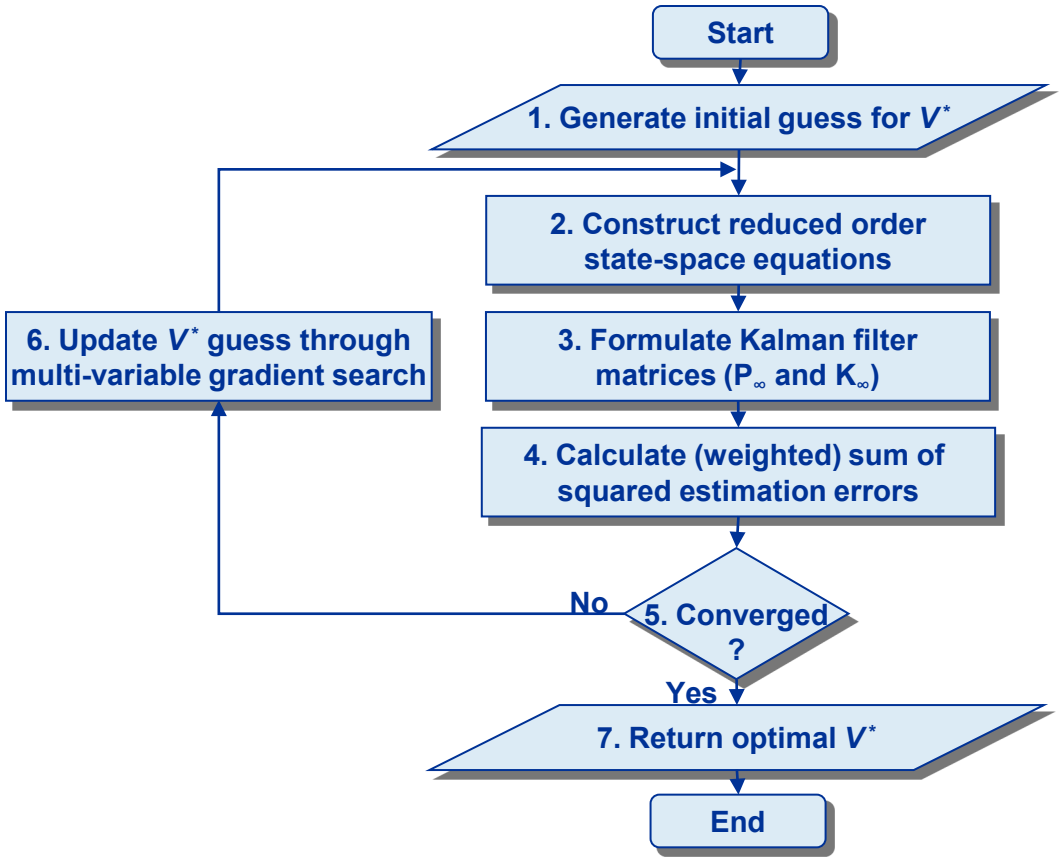
$$SSEE_{\hat{x},\hat{h}} = trace\left\{G_{xh}P_hG^T_{xh} + P_{\hat{xh},k}\right\} \qquad WSSEE_{\hat{z}} = tr\left\{W_z\left[G_zP_hG^T_z + P_{\hat{z},k}\right]\right\}$$

Optimization problem: Choose V^* to minimize SSEE / WSSEE in the parameters of interest

1) Minimization of the Kalman Filter Mean Squared Estimation Error Under Closed-Loop Operating Conditions

Optimal Iterative Search for V^*

- Performed using the Matlab *lsqnonlin* function
- Conducted off-line during Kalman filter design stage
- Does not return a unique V^* each time executed, but mean sum of squared estimation errors is very consistent
- V^* returned from optimal iterative search defines optimal tuner vector: $q = V^* h$



Optimal iterative search flow chart

2) Selection of "Globally Optimal" Model-Tuner Vector

- Conventional design approach for on-board adaptive models is to implement a piecewise linear Kalman filter with a fixed $q \subseteq h$
  - Design individual Kalman filters at multiple operating points.
  - Interpolate between points as engine transitions through operating envelope.

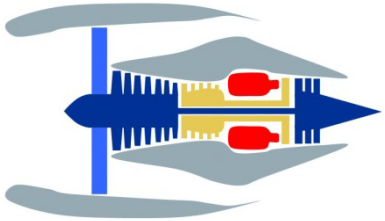
- If a different V^* (and tuner vector, q) is selected at each operating point comprising the piecewise linear design, interpolation between points will yield meaningless results.

- Solution: Modify optimal iterative search to minimize the mean sum of squared estimation errors at multiple user-defined operating points instead of a single operating point.
  - Adds computational complexity during the design process, but on-line computational complexity is the same.

Turbofan Engine Simulation Example

The enhanced model tuner selection strategy was applied to NASA Commercial Modular Aero-Propulsion System Simulation (C-MAPSS)

- Non-linear high bypass turbofan engine simulation with control system



Health Parameters *h*	Sensed outputs (y)	Auxiliary parameters (z)
FAN efficiency	Nf – fan speed (rpm)	T40 – Combustor exit temp. (°R)
FAN flow capacity	Nc – core speed (rpm)	T50 – LPT exit temperature (°R)
LPC efficiency	P24 – HPC inlet total pressure (psia)	Fn – Net thrust (%)
LPC flow capacity	T24 – HPC inlet total temp. (°R)	SmLPC – LPC stall margin (%)
HPC efficiency	Ps30 – HPC exit static pressure (psia)	
HPC flow capacity	T48 – Exhaust gas temperature (°R)	
HPT efficiency		
HPT flow capacity		
LPT efficiency		
LPT flow capacity		

Turbofan Engine Example (cont.)

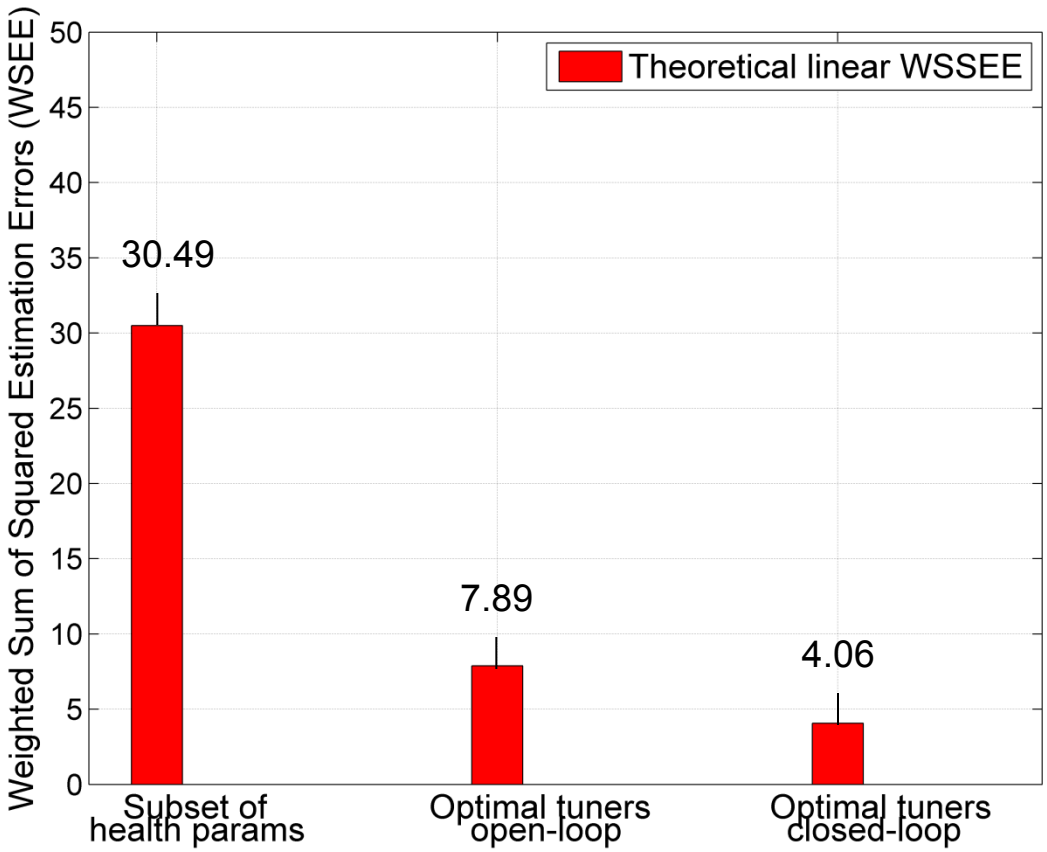
Case 1: Kalman filter point design at a cruise operating point (35K feet, 0.65 Mach, 60° PLA)

- Designed Kalman filters applying the following three approaches:
  - Tuners selected as a subset of health parameters (conventional approach)
  - Optimal tuner selection methodology (open-loop operation)
  - Optimal tuner selection methodology (closed-loop operation)
- Calculated theoretical linear mean sum of squared estimation errors
- Conducted linear (linear point model) and non-linear (full nonlinear C-MAPSS) Monte Carlo simulation studies to experimentally determine mean sum of squared estimation errors

Turbofan Engine Example

Case 1: Auxiliary Parameter Estimation Results at Cruise

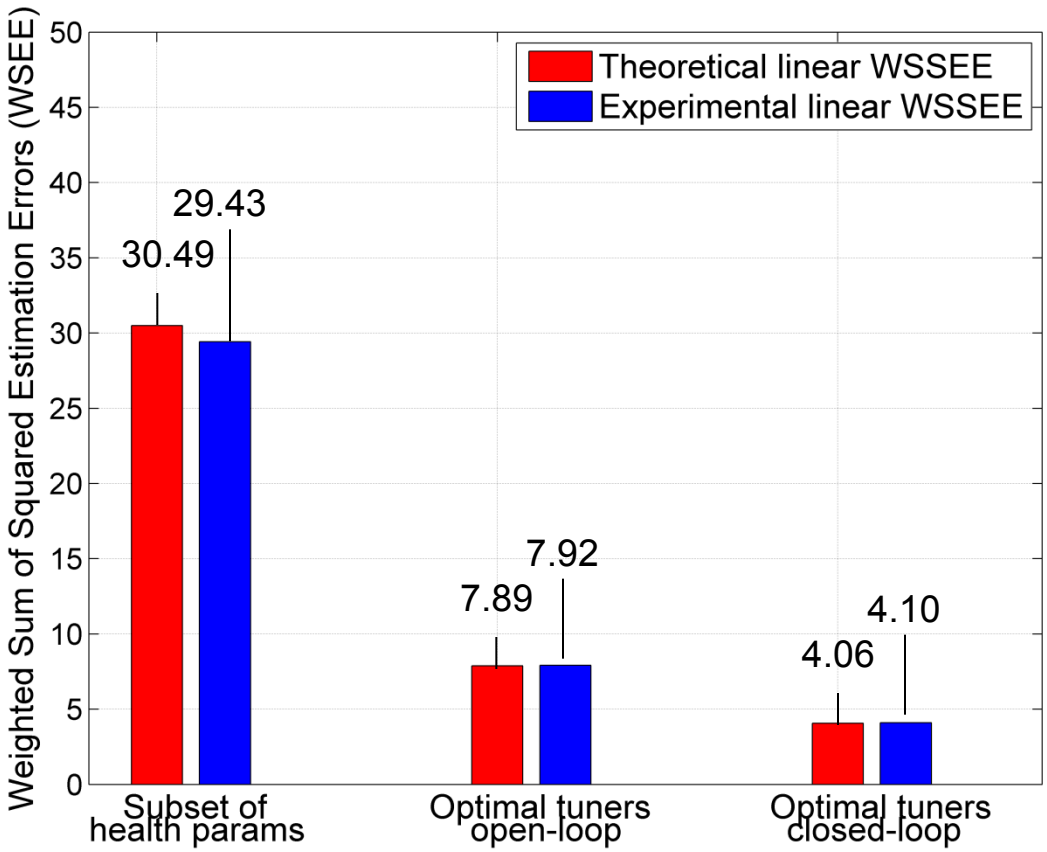
Auxiliary parameter mean sum of squared estimation errors **(Theoretical linear WSSEE)**



Turbofan Engine Example

Case 1: Auxiliary Parameter Estimation Results at Cruise

Auxiliary parameter mean sum of squared estimation errors **(Theoretical and experimental linear WSSEE)**

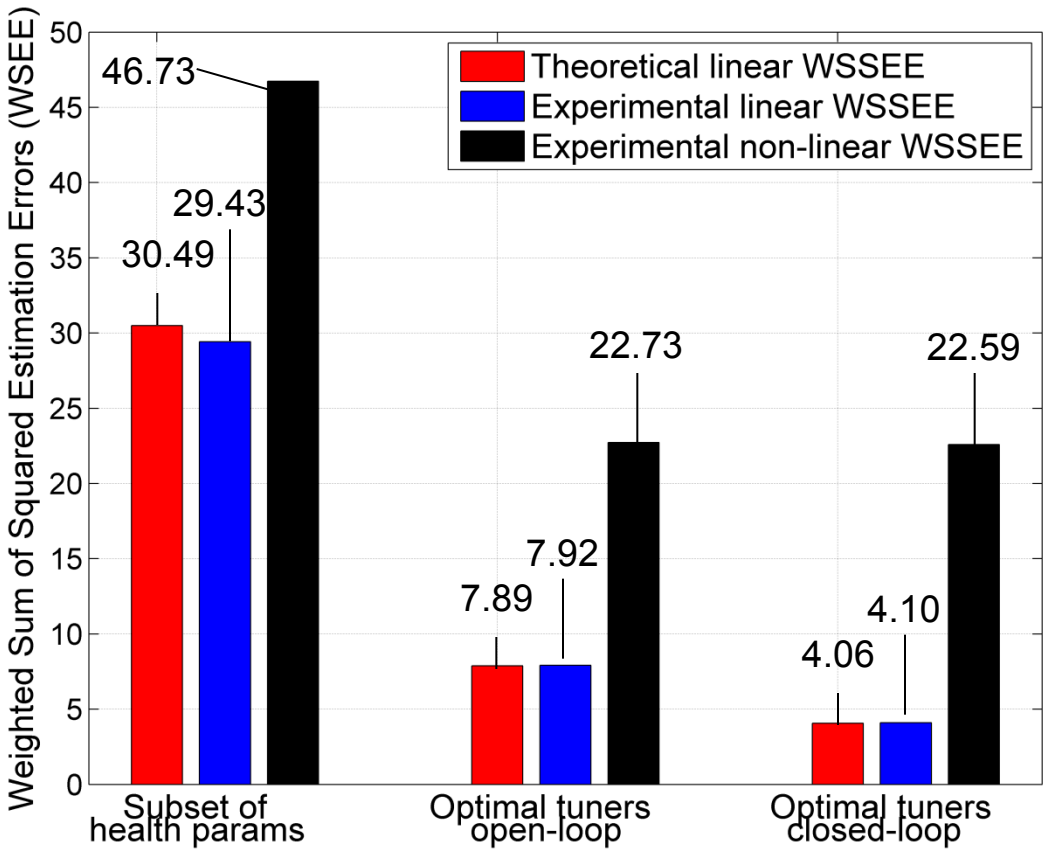


Turbofan Engine Example

Case 1: Auxiliary Parameter Estimation Results at Cruise

Auxiliary parameter mean sum of squared estimation errors

(Theoretical, experimental linear, and experimental nonlinear WSSEE)



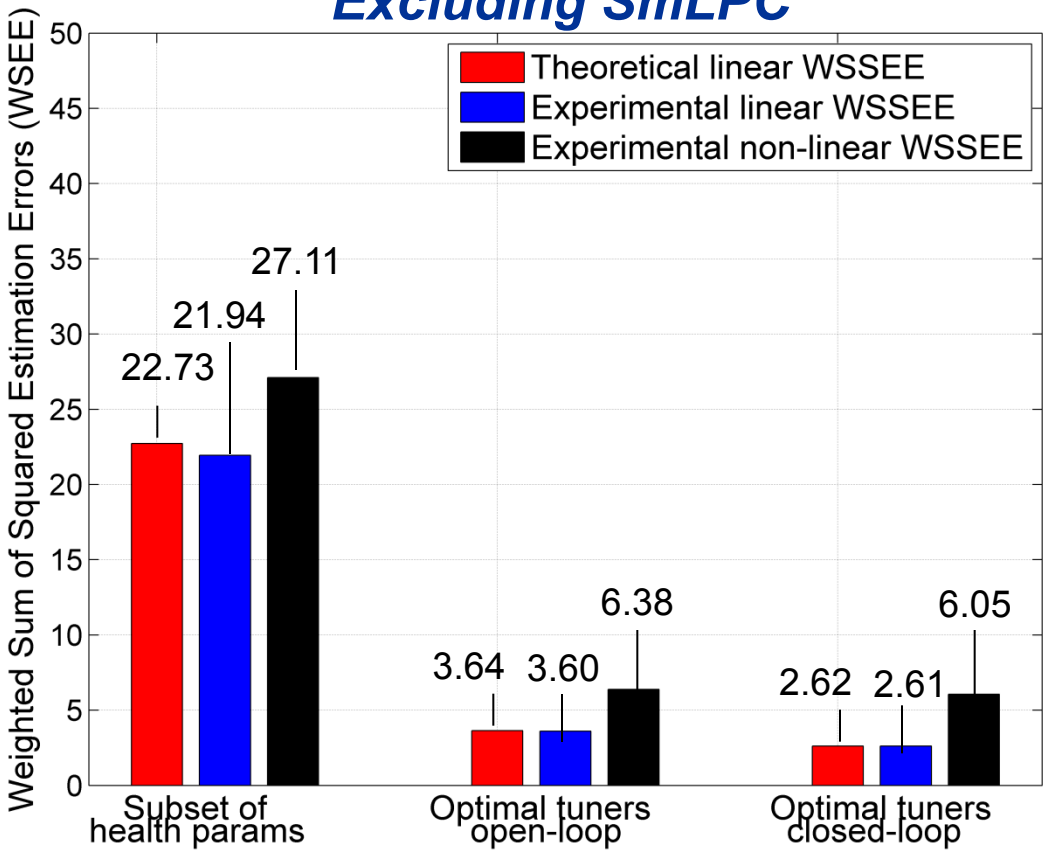
Turbofan Engine Example

Case 1: Auxiliary Parameter Estimation Results at Cruise

Auxiliary parameter mean sum of squared estimation errors

(Theoretical, experimental linear, and experimental nonlinear WSSEE)

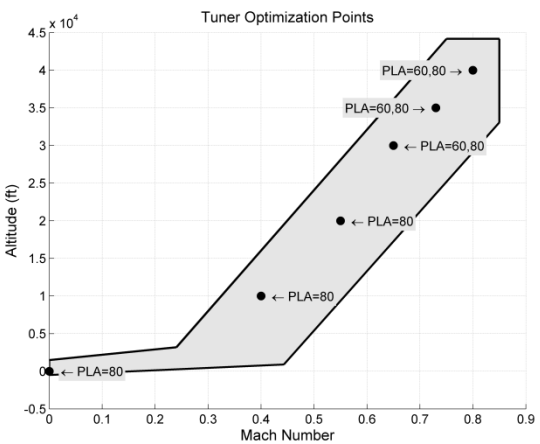
Excluding SmLPC



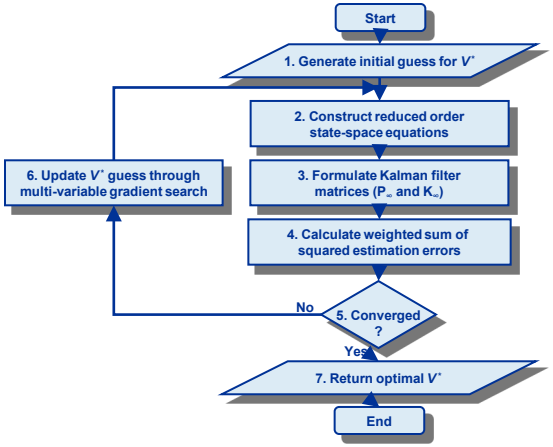
Turbofan Engine Example (cont.)

Case 2: Kalman filter full-envelope design

- Designed Kalman filters applying the following four approaches:
  - Tuners selected as a subset of health parameters (conventional approach)
  - Optimal tuner selection methodology (open-loop operation)
  - Optimal tuner selection methodology (closed-loop operation)
  - Closed-loop globally optimal tuner selection based on nine operating points spanning commonly encountered engine operating points.



Nine operating points evaluated in globally optimal tuner selection example

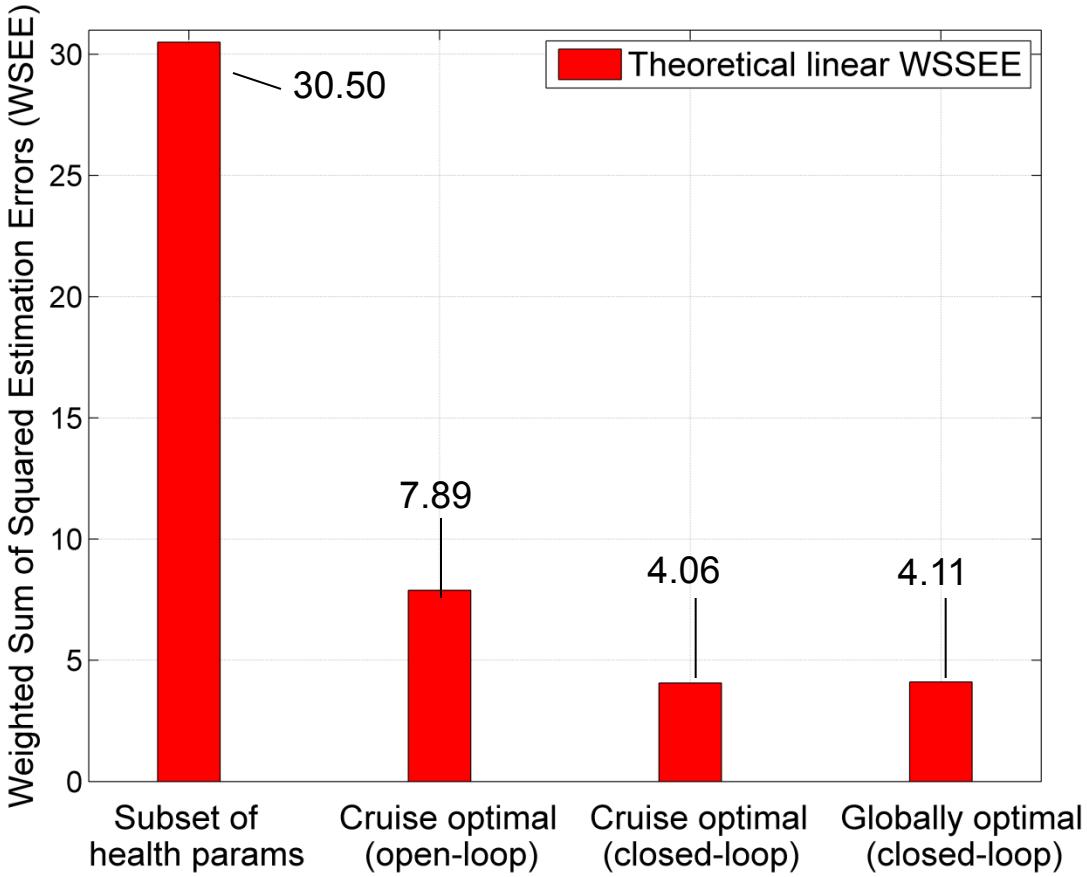


Optimal Iterative Search Modified to Minimize Error Over Multiple Operating Points

Turbofan Engine Example

Case 2: Auxiliary parameter estimation results

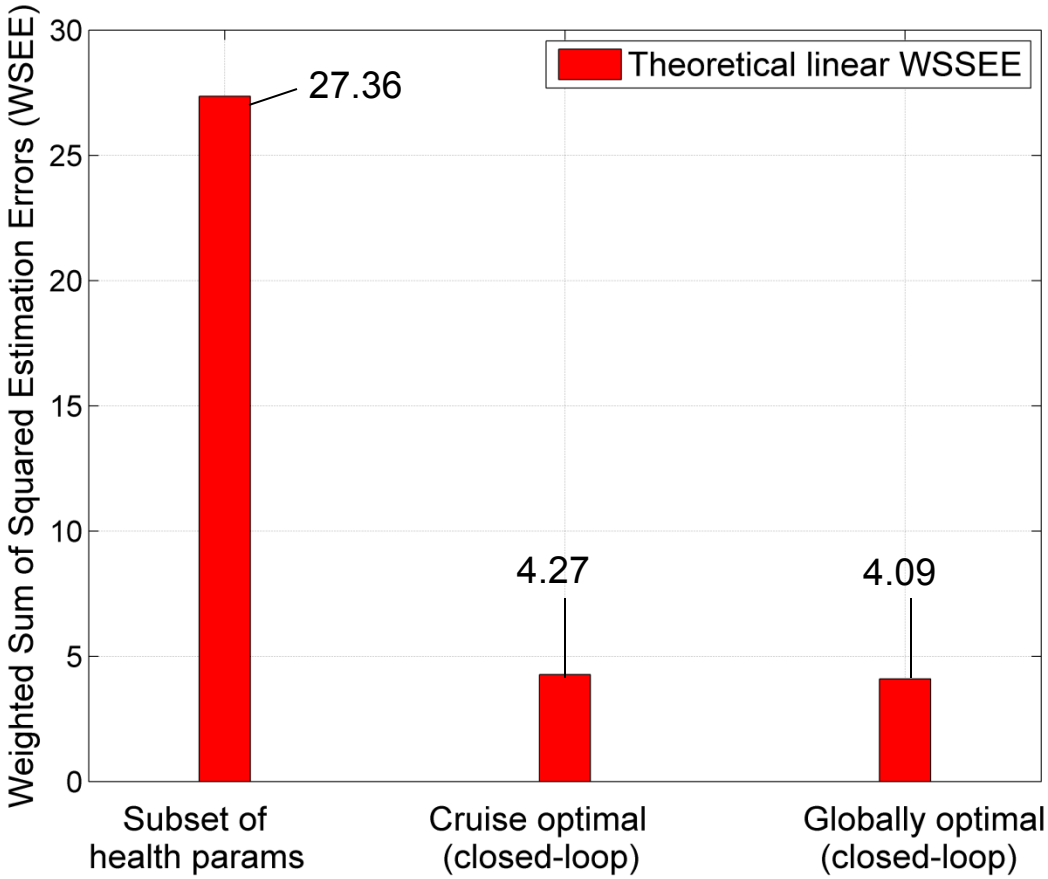
Auxiliary parameter theoretical mean sum of squared estimation errors at steady-state cruise operating point (35K feet, 0.65 Mach, 60° PLA)



Turbofan Engine Example

Case 2: Auxiliary parameter estimation results

Auxiliary parameter theoretical mean sum of squared estimation errors averaged over 2000 steady-state operating points



Conclusions

- Extended optimal tuner selection methodology for closed-loop operating conditions. Application to C-MAPSS revealed:
  - Estimation accuracy significantly better than conventional approach of selecting a subset of health parameters
  - But found that overall estimation accuracy is only slightly superior to open-loop optimal tuner selection methodology
- Developed and evaluated globally optimal tuner selection
  - Found the loss in estimation accuracy versus optimal point design is minimal
- Follow-on work:
  - Incorporate a direct measure of Kalman filter dynamics within optimal iterative search process
  - Conduct evaluations using actual engine data
  - Apply within model-based engine control applications

Model-Based Aircraft Engine Performance Estimation

Publications

Conference papers/presentations:

- Donald L. Simon, Sanjay Garg (2009), "A Systematic Approach for Model-Based Aircraft Engine Performance Estimation," AIAA Infotech@Aerospace Conference, Apr. 6-9, 2009 Seattle, WA.
- Donald L. Simon, Sanjay Garg (2009), "Optimal Tuner Selection for Kalman Filter-Based Aircraft Engine Performance Estimation," GT2009-59684, ASME Turbo Expo Conference, Jun. 8-12, 2009 Orlando, FL.
- Donald L. Simon, Sanjay Garg (2009), "A Systematic Approach to Sensor Selection for Aircraft Engine Health Estimation," ISABE-209-1125, International Society of Air Breathing Engines Conference, Sep. 7-11, 2009 Montreal, QC, Canada.
- Donald L. Simon, Jeffrey B. Armstrong, Sanjay Garg (2011), "Application of an Optimal Tuner Selection Approach for On-Board Self-Tuning Engine Models," GT2011-46408, ASME Turbo Expo Conference, Jun. 6-10, 2011 Vancouver, BC, Canada.

Journal articles:

- Donald L. Simon, Sanjay Garg(2009), "Optimal Tuner Selection for Kalman-Filter Based Aircraft Engine Performance Estimation," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 132 / 031601-1-10 (March 2010).
- Donald L. Simon, Jeffrey B. Armstrong, Sanjay Garg (2011), "Application of an Optimal Tuner Selection Approach for On-Board Self-Tuning Engine Models," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 134 / 041601-1-11, (April 2012)