Atmospheric Turbulence Modeling and Shock Positioning Control for a Supersonic Inlet

Fundamental Aeronautics – Supersonics Project
AeroPropulsoServoElasticity Task

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Atmospheric Turbulence Modeling for High Speed Vehicles

Outline

Motivation/Background

Atmospheric Turbulence Model Development

-- Approach

-- Formulations

-- Examples (Spectral Densities & Time Domain)

Concluding Remarks

Atmospheric Turbulence Modeling for High Speed Vehicles

Motivation

-- Atmospheric turbulence modeling needed for both propulsion and flight control designs

-- Atmospheric Turbulence is fractional order (5/3 order)

-- Current 2nd order model approximation used deviates (underestimates) up to 7dB/decade (i.e. 20db x 2 vs. 20dB x 5/3)

Atmospheric Turbulence – Power Spectral

-- Atmospheric models primarily based on the "Kolmogorov Spectral", developed by Taraski, 1961, based partially on studies by Kolmogorov, 1941.

$$S_t(k) = \alpha_t \varepsilon^{2/3} k^{-5/3}$$

α – constant for the type of disturbance
ε – eddy dissipation rate (energy/(mass * time))
k – wave number (rad/m)

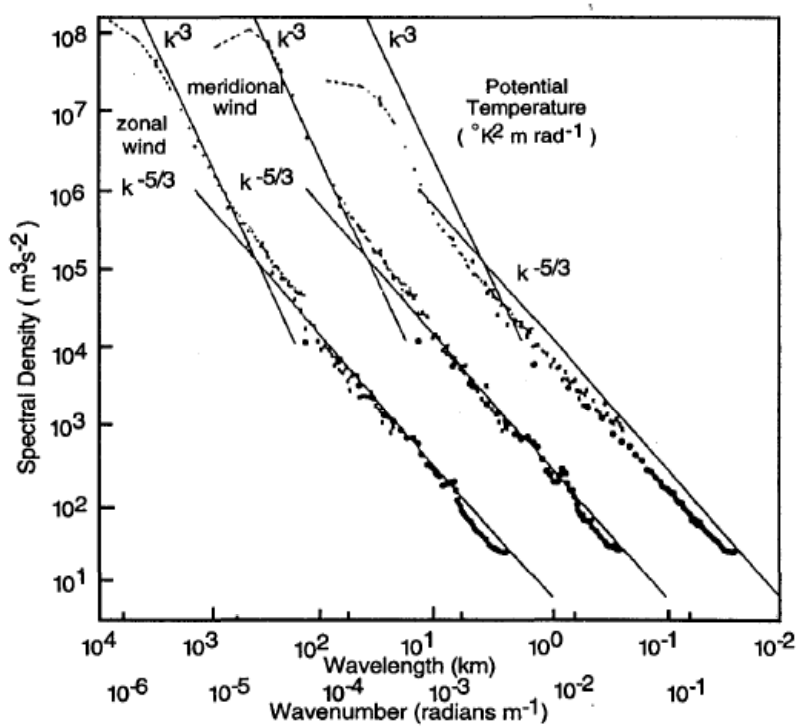


Figure – Wind and Potential Temperature Spectra as reported by Nastrom (1985). Note: for clarity, the meridional wind and temperature spectra have been shifted one and two decades to the right, respectively.

Atmospheric Turbulence – Power Spectral

-- Kolmogorov Spectral has infinite energy – increases in magnitude with decreasing frequency

-- Tank, 1994 scaled the von Karman spectrum to fit the Kolmogorov model

$$S_{l,VK}(k) = 2.7\varepsilon^{2/3} L^{5/3} \frac{2}{[1+(1.339(2\pi)Lk)^2]^{5/6}}$$ longitudinal disturbance

$$S_{v,VK}(k) = 2.7\varepsilon^{2/3} L^{5/3} \frac{1+\frac{8}{3}(1.339(2\pi)Lk)^2}{[1+(1.339(2\pi)Lk)^2]^{11/6}}$$ transverse disturbance

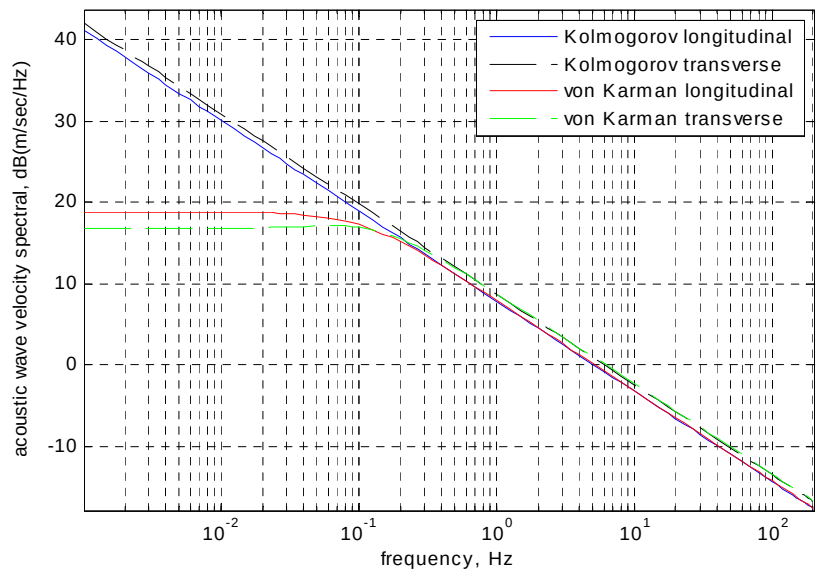


Figure – Acoustic wave velocity spectral comparisons for the Kolmogorov and Von Karman spectral

Atmospheric Turbulence – Power Spectral Solution Challenges

-- Atmospheric disturbances are fractional order

-- Solution of fractional order derivatives are difficult $\longrightarrow$ abbey the law of non-locality $\longrightarrow$ state transition matrix is a convolution integral

-- To get around this problem Hoblit, 1998 introduced the Dryden model

$$\Phi(\Omega)_u = \frac{2\sigma^2 L_u}{\pi} \frac{1}{1+(L_u \Omega)^2}$$

-- This model is 2nd order instead of 5/3 order, increasingly deviates with frequency (by 7db/decade)

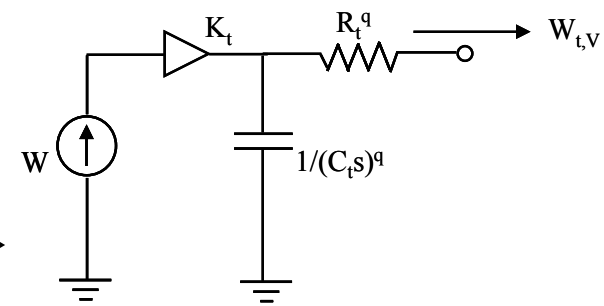
-- Based on that the idea was born to develop our own fractional order model

Atmospheric Turbulence – Fractional Order Model Development

1) $$S_{l,VK}(k) = 2.7\varepsilon^{2/3} L^{5/3} \frac{2}{[1+(1.339(2\pi)Lk)^2]^{5/6}}$$

$$S_{l,VK}(k) = \left(2.7\varepsilon^{2/3} L^{5/3} \frac{2}{[1+(1.339L\frac{2\pi f}{Ma_o})^2]^{5/6}} \right)^{1/3}$$

$$S_t(k) = \left(\alpha_t \varepsilon^{2/3} \frac{f}{Ma}^{-5/3} \right)^{1/3}$$



Circuit Equivalent of Atmospheric Disturbances

2) Formulate circuit parameters in terms of vehicle speed (f= kMa) and atmospheric disturbance parameters

$$C_t = \frac{1}{(a_t\varepsilon^{2/3})^{1/x}(2\pi Ma_o)}, \quad R_t = 1.339(2\pi)(a_t\varepsilon^{2/3})^{1/x} L, \quad \omega_f = \frac{K_{\omega n}}{R_t C_t}, \quad K_t = f(\varepsilon, L)$$

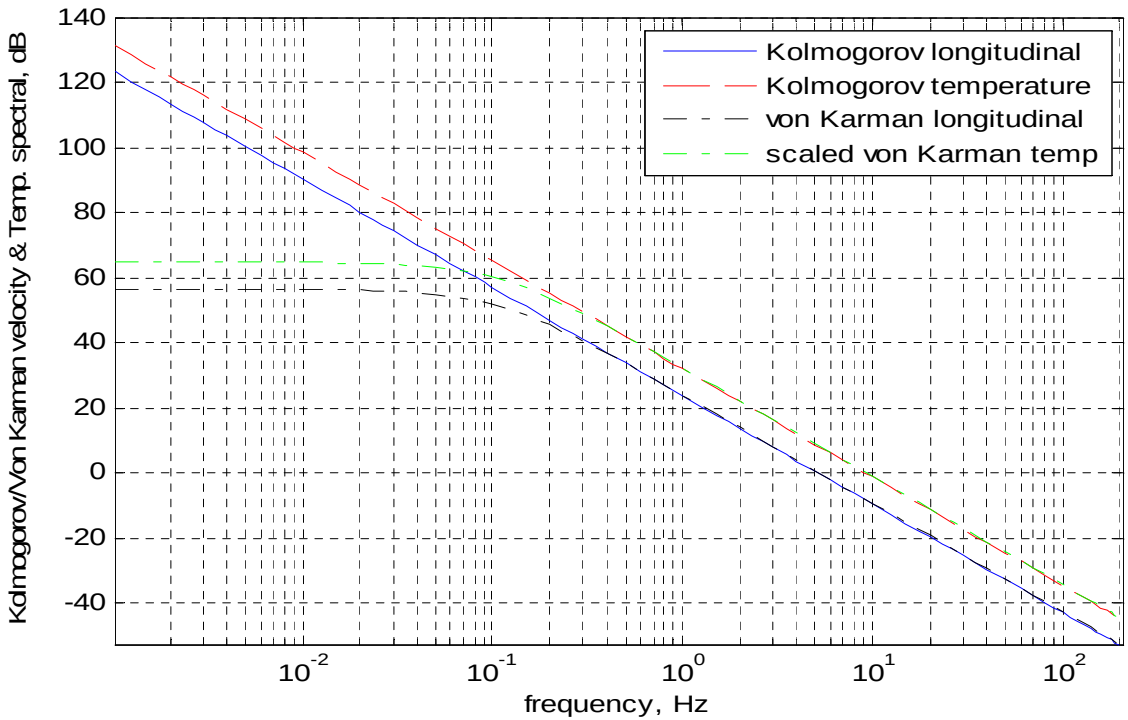
3) Develop von Karman temperature, pressure and density spectral by scaling the respective Kolmogorov spectral to fit the von Karman spectrum for wind gusts.

4) Plot von Karman Spectral and using circuit formulations, formulate a series of first order pole-zero TF to approximate fractional order TF.

Atmospheric Turbulence – Fractional Order Model Development

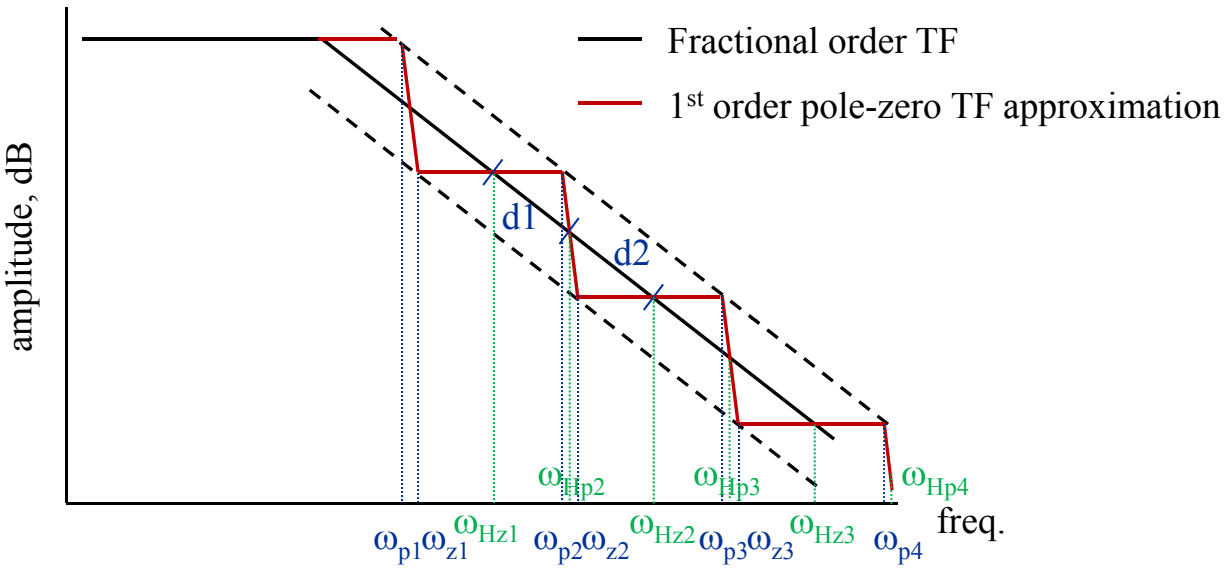
Step 3. scaling for temperature

$$S_t(k) = \left(\alpha_t \varepsilon^{2/3} \frac{f}{Ma}^{-5/3} \right)^{1/3}$$



Atmospheric Turbulence – Fractional Order Model Development

Step 4. formulation of pole-zero TF approach



-- Formulate 1st order pole-zero TF approximation subject to

(a) $\omega_{pi,\ zi}=f(\omega_{p1}..\omega_{pi-1},\omega_{z1}..\omega_{zi-1})$

(b) Equal distance: d1=d2=..(symmetry) ⇨ $\omega_{pi,\ zi}=f(\omega_{p1}..\omega_{pi-1},\ \omega_{z1}..\omega_{zi-1},\ \omega_{Hpi,}\ \omega_{Hzi})$

(c) ω=f(circuit parameters)

(d) TF also function of desirable pole-zero density per decade ⟶ closer approximation

Atmospheric Turbulence – Fractional Order Model Development

-- Based on this approach the following formulations are derived

$$W_{t,o} \cong K_{t,fit} \frac{\prod_{1}^{m_z}(s/\omega_{zi}+1)}{\prod_{1}^{m_p}(s/\omega_{pi}+1)} W_t$$

Generalized atmospheric disturbance TF

$$\omega_{pi} = \frac{K_{\omega pi}\omega_{Hpi}\prod_{j=1}^{i-1}(\omega_{Hpi}/\omega_{pi-j}+1)}{10^{\eta(2i-1)*q}\prod_{j=1}^{i-1}(\omega_{Hpi}/\omega_{zi-j}+1)-1},$$

$$\omega_{Hpi} = \omega_f (10^{\eta q(2i-1)}-1)^{1/q}$$

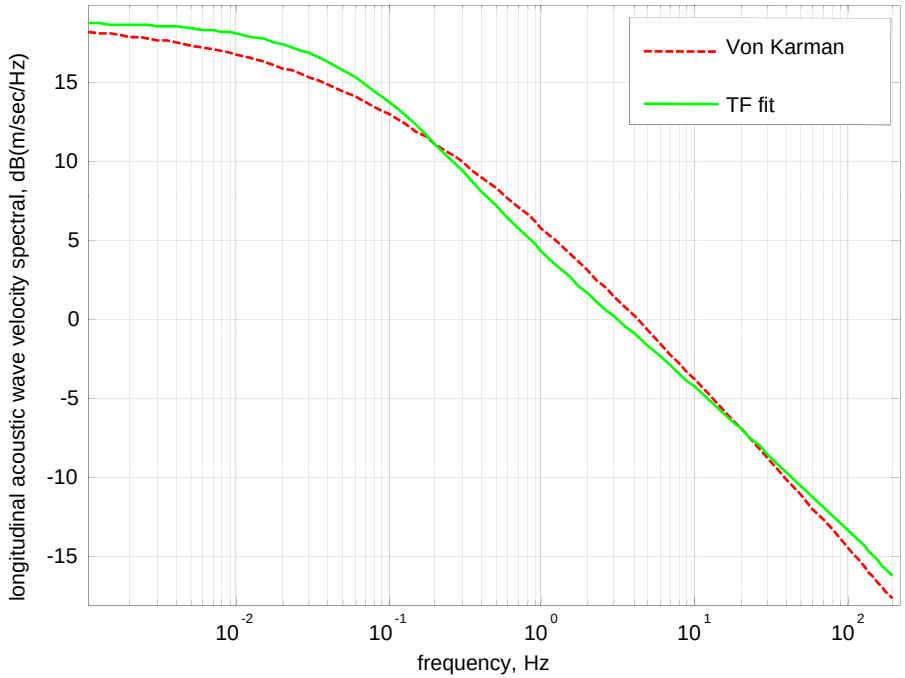
Pole-zero frequency computations

$$\omega_{zi} = \frac{K_{\omega zi}\omega_{Hzi}\prod_{j=1}^{i-1}(\omega_{Hzi}/\omega_{zi-j}+1)}{10^{-2\eta i*q}\prod_{j=1}^{i}(\omega_{Hzi}/\omega_{pi}+1)-1},$$

$$\omega_{Hzi} = \omega_f (10^{2\eta q i}-1)^{1/q}$$

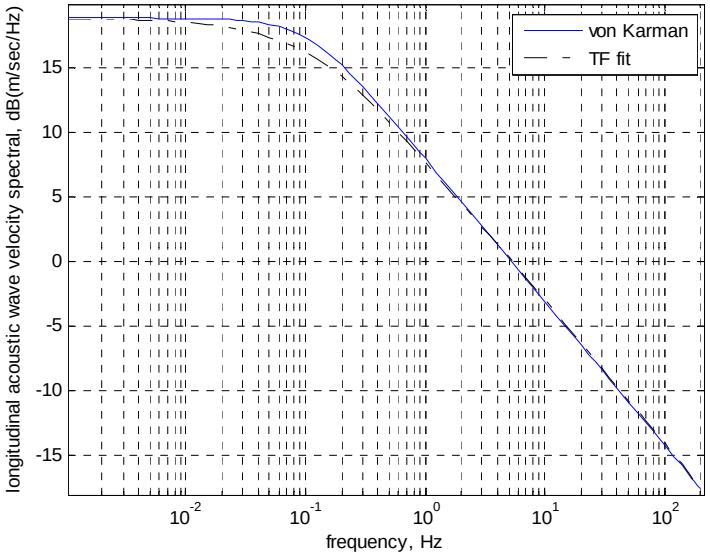
Atmospheric Turbulence – Fractional Order Model Development

-- Example fit using equations with certain atmospheric parameters:

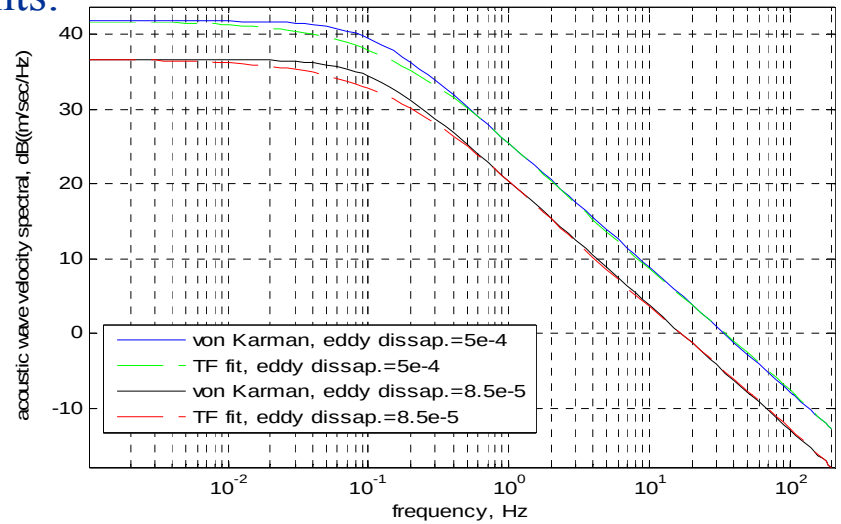


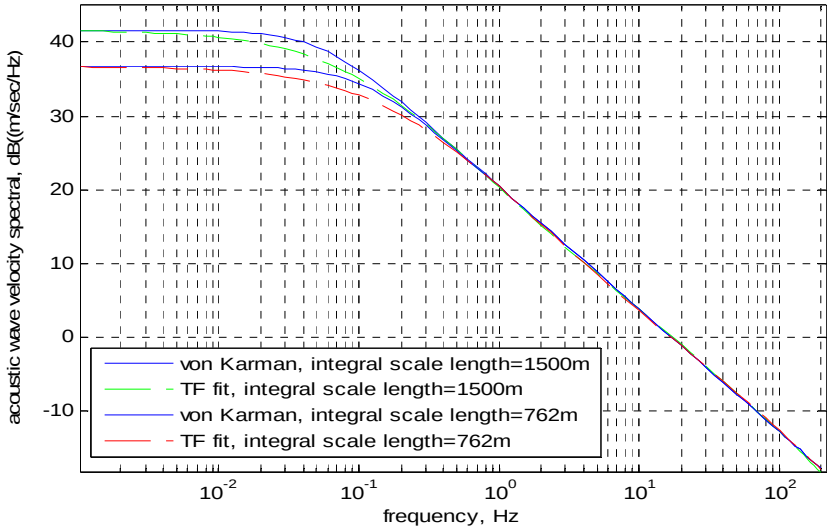
Atmospheric Turbulence – Fractional Order Model Development

-- Example fits using these Eqs. w/ adjustments:



Longitudinal Acoustic wave velocity spectral





Acoustic wave velocity spectral due to temperature gust and its TF fits

Atmospheric Turbulence – Fractional Order Model Development

-- TF fits with L=765, ε=8.6e-5):
(complete equation set takes into account these adjustments)

$$G_{LA}(s) = \frac{8.74(s/9.2+1)(s/55.0+1)(s/335.5+1)}{(s/1.46+1)(s/30.1+1)(s/85.7+1)(s/1593.1+1)}$$

Longitudinal Acoustic Wave Velocity (m/sec)

$$G_T(s) = \frac{41.75(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

Temperature (K)

$$G_{TA}(s) = \frac{M\gamma R}{a_o}\frac{20.88(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

Acoustic Wave due to Temperature (m/sec)

$$G_P(s) = \frac{37.96(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

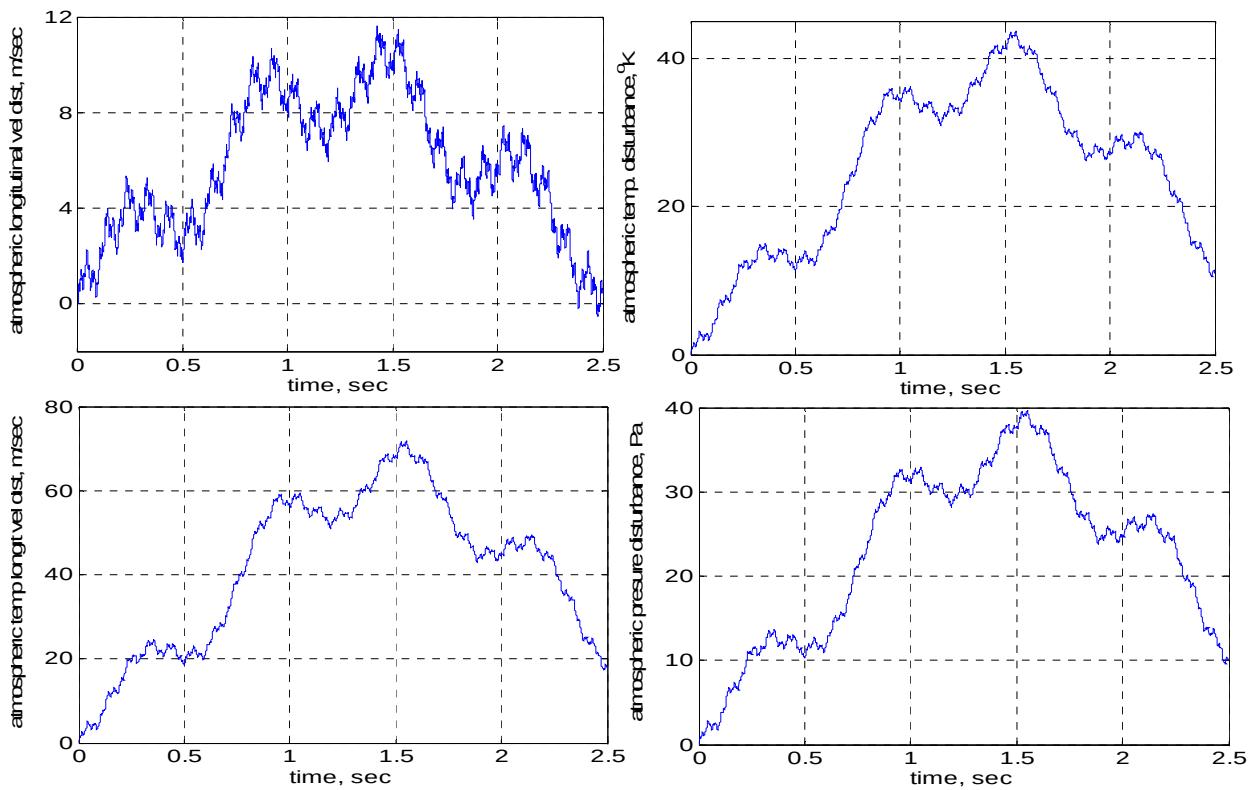
Pressure (Pa)

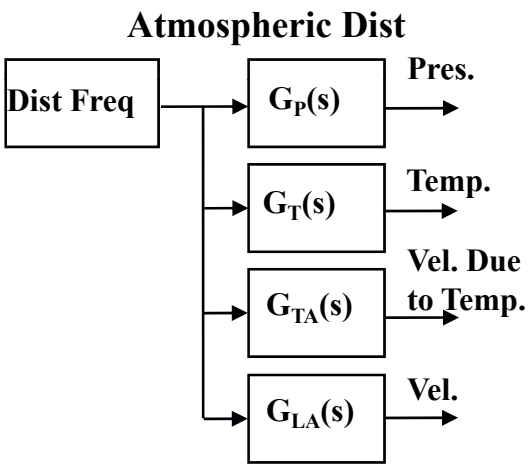
Atmospheric Turbulence – Fractional Order Model Development

-- Simulating atmospheric Disturbances:

a) Construct a time domain unit amplitude sinusoids distributed in frequency

b) Use this time domain sinusoids as input to atmospheric disturbance TFs





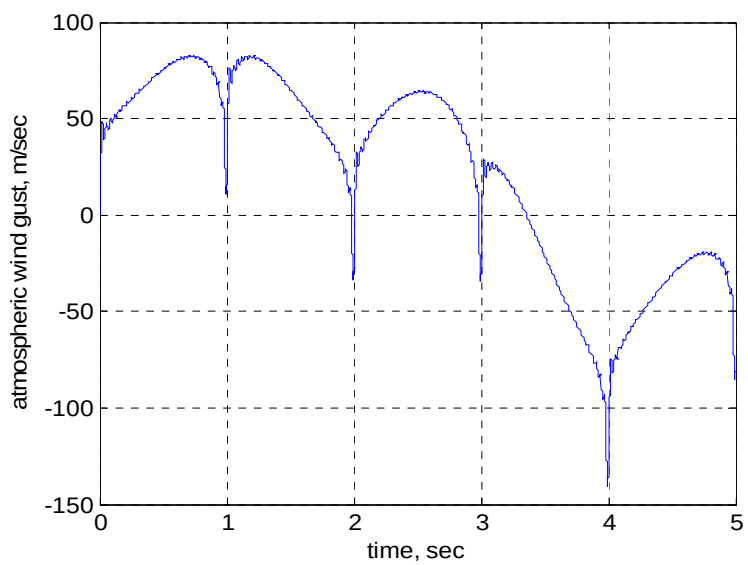
Atmospheric Disturbances w/ frequencies from sub Hz to 200 Hz, w/ ε=8.6e-5, L=765 m (11km turbulence patch)

Atmospheric Turbulence – Fractional Order Model Development

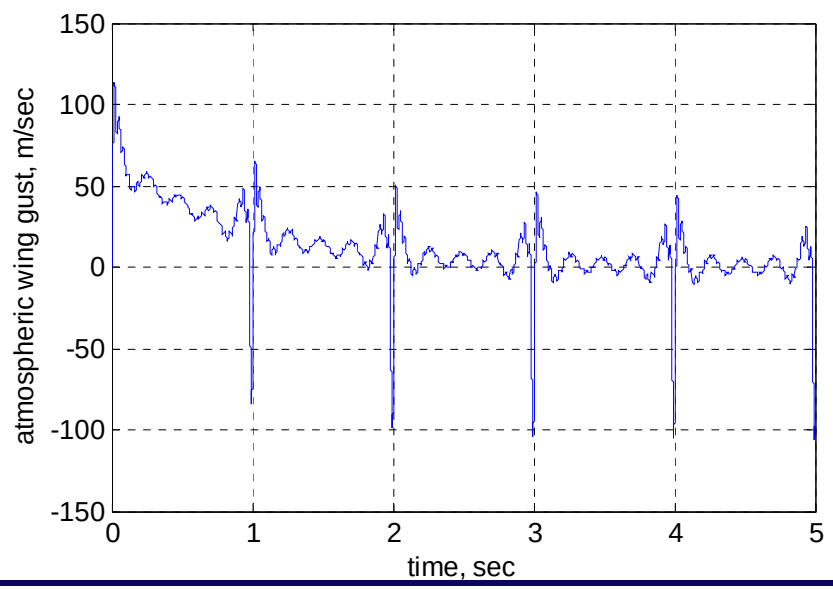
-- Atmospheric wind gust turbulence considering:
- -- Vehicle Mach=2.35
- -- Smallest eddy dissipation scales of 25m (from literature)
- -- ~180 mph top wind speeds
- -- Eddy dissipation rate

-- Results in cluster frequencies up to 30 Hz

Light to Moderate Turbulence (ε=8.6e-5)



Moderate to Severe Turbulence (ε=1.7e-3)



Future Work

-- Come up with considerations for atmospheric disturbance specifications for high speed vehicles

-- Integrated Propulsion system studies utilizing atmospheric disturbances

-- Integrated Vehicle (Aero-Propulso-Servo-Elastic) studies utilizing atmospheric and other disturbances

Publications:

1. Kopasakis, "Atmospheric Turbulence Modeling for Aero Vehicles – Fractional Order Fits," NASA TM, in process of publishing.

2. Kopasakis, "Modeling of Atmospheric Turbulence as Disturbances for Control Design and Evaluation of High Speed Propulsion Systems," ASME TurboExpo 2010 to be published

Shock Positioning Controls Design for a Supersonics Inlet

Outline

Background

Controls Design Approach

Inlet Shock Positioning Controls Design

-- Plant Transfer Functions

-- Atmospheric Disturbances

-- Controls Design

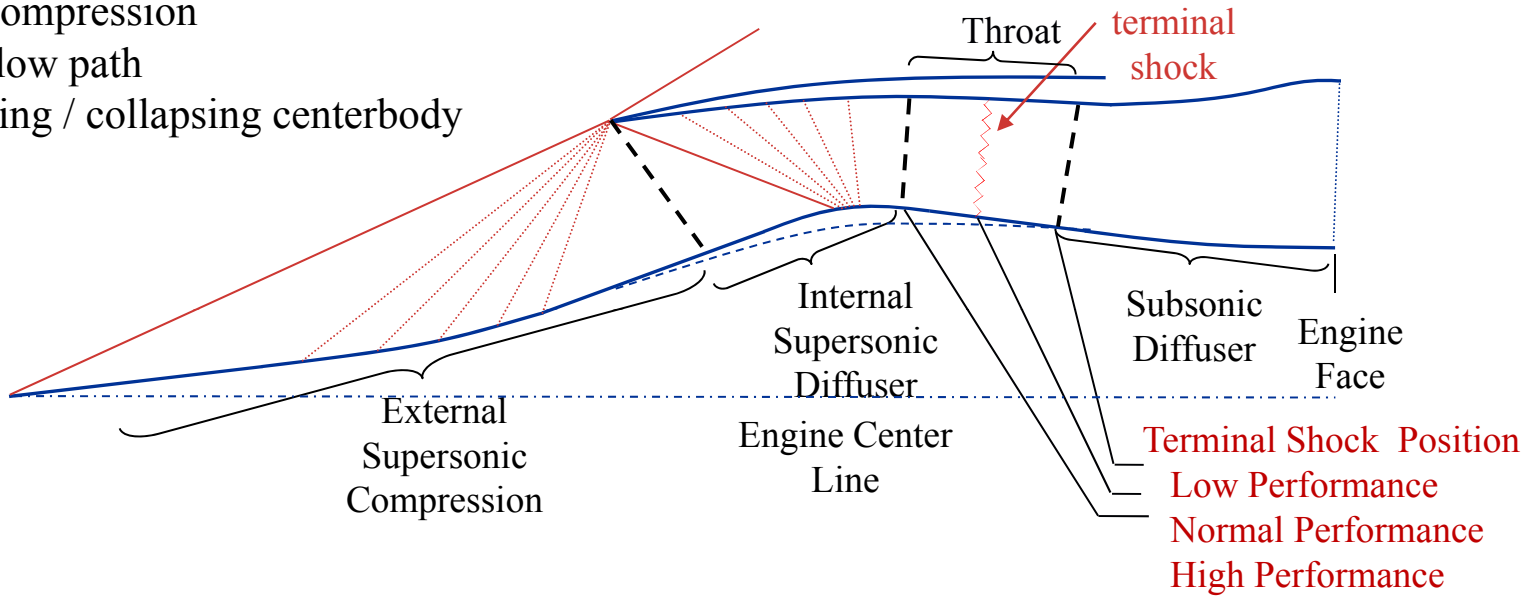
-- Results

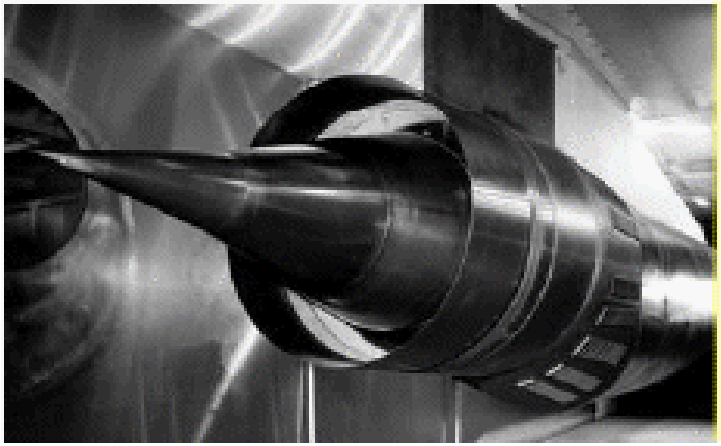
Concluding Remarks

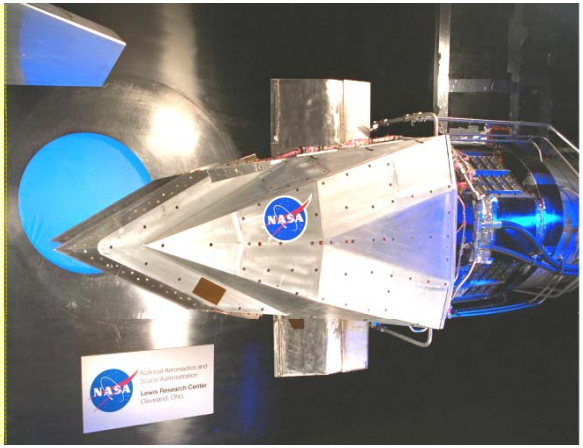
Shock Positioning Controls Design for a Supersonics Inlet

Background

Supersonic Axisymmetric Inlet
- Mixed-compression
- Single flow path
- Translating / collapsing centerbody







Feedback Controls Loop Shaping Design Approach

Motivation:

- Tie controls design to hardware capabilities.

- Maximize performance
  - -- Better command tracking
  - -- Enabling reduction of design margins to improve propulsion efficiency.

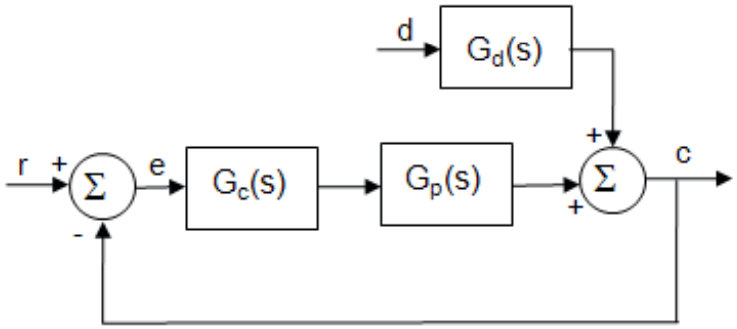
Features of Approach:

- Using control system requirements and actuator speed(s) to shape the feedback controller Closed Loop Gain (CLG)

- Methodical design w/ quantifiable performance
  - -- Emphasizes how to trade competing requirements
  - -- Maximize performance (command tracking, disturbance rejection, stability
  - -- Guaranteed stability by avoiding plant inversion

Kopasakis, "Feedback Control Systems Loop Shaping Design with Practical Considerations," NASA/TM-2007-215007

Feedback Controls Loop Shaping Design Approach



Feedback Control System Diagram.

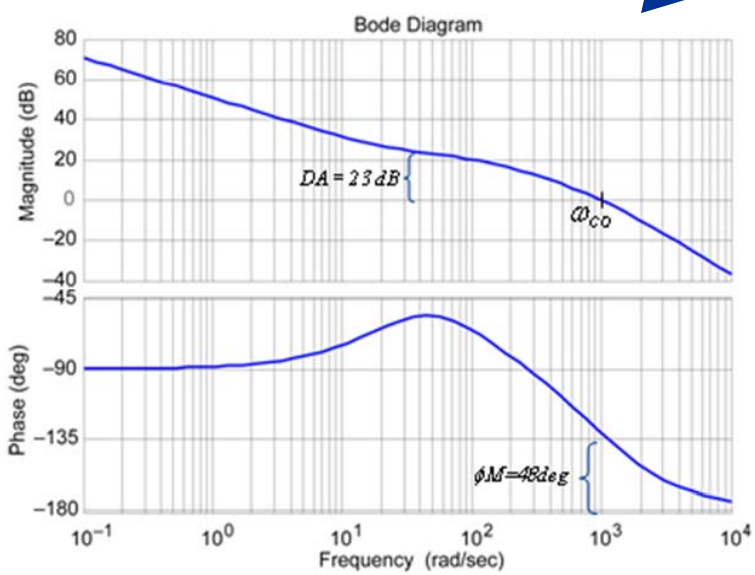
$$\frac{C(s)}{R(s)} = \frac{L(s)}{1+L(s)}, \qquad \frac{C(s)}{D(s)} = \frac{G_d(s)}{1+L(s)}$$

Step a. – Select CLG bandwidth,

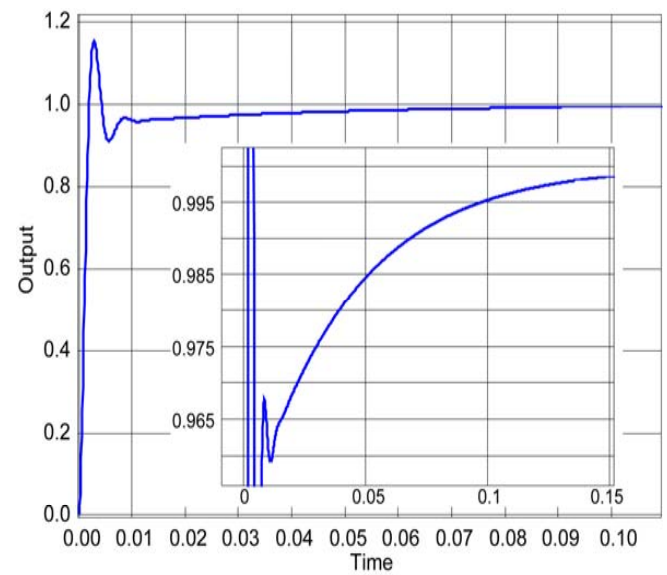
$$\omega_{co} = C_r a_{rl} / r_m$$

Step b. – Design desired CLG for specifications (stability, disturbance rejection, etc.)

$$L(s) = 353.13 \frac{(s/25+1)}{s(s/100+1)(s/1000+1)}$$



Example of Closed Loop Gain (CLG) Design



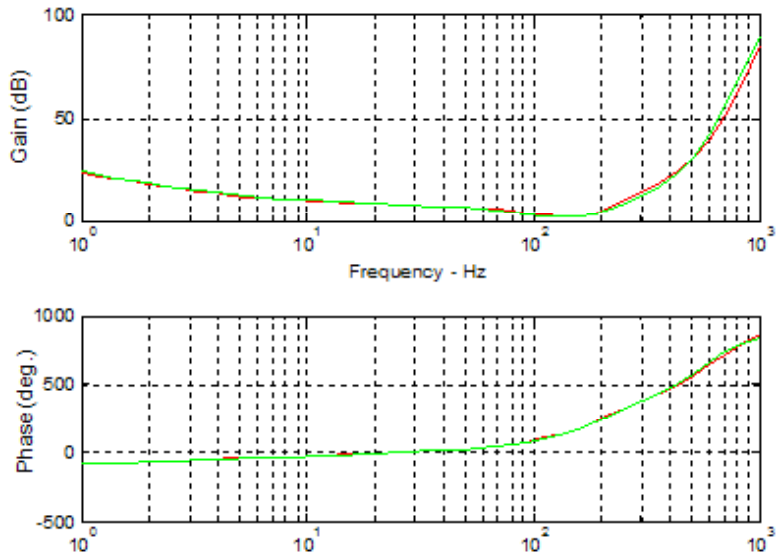
Step Response

Feedback Controls Loop Shaping Design Approach

Step c. – Calculate control TF and plot

$$|G_C(s)| = 20\log_{10}|L(s)| - 20\log_{10}|G_P(s)|$$

$$\phi(G_C(s)) = \phi(L(s)) - \phi(G_P(s))$$

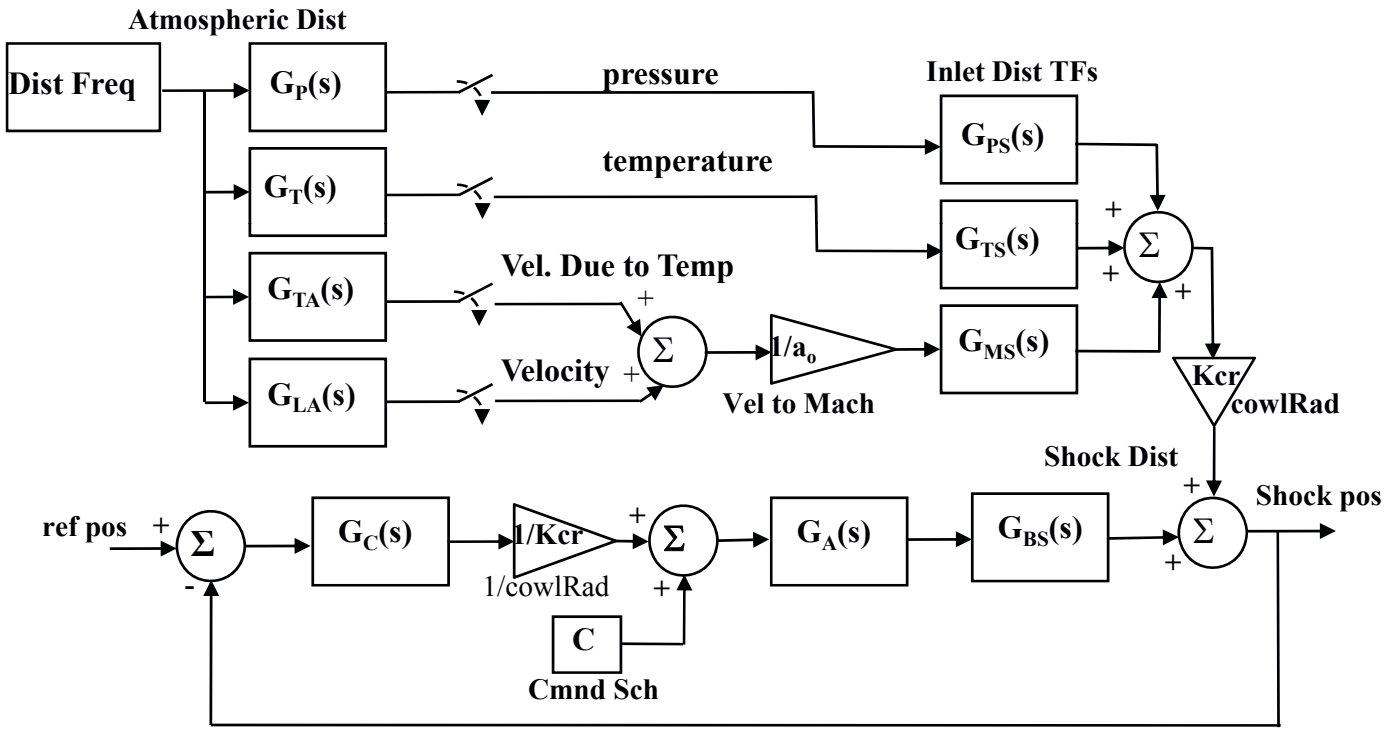


Step d. – Fit control TF w/ poles and zeros

$$G_C(s) = \frac{K(s/\omega_{z1}+1)....(s^2/w_{zn}^2 + 2\varsigma/\omega_{zn}+1)}{s(s/\omega_{p1}+1).....(s^2/w_{pn}^2 + 2\varsigma/\omega_{pn}+1)}$$

Inlet Shock Position Controls Design

-- Control the opening of bypass doors to position the shock against disturbances (atmospheric turbulence, aeroservoelastic, pitch, yaw, and engine)



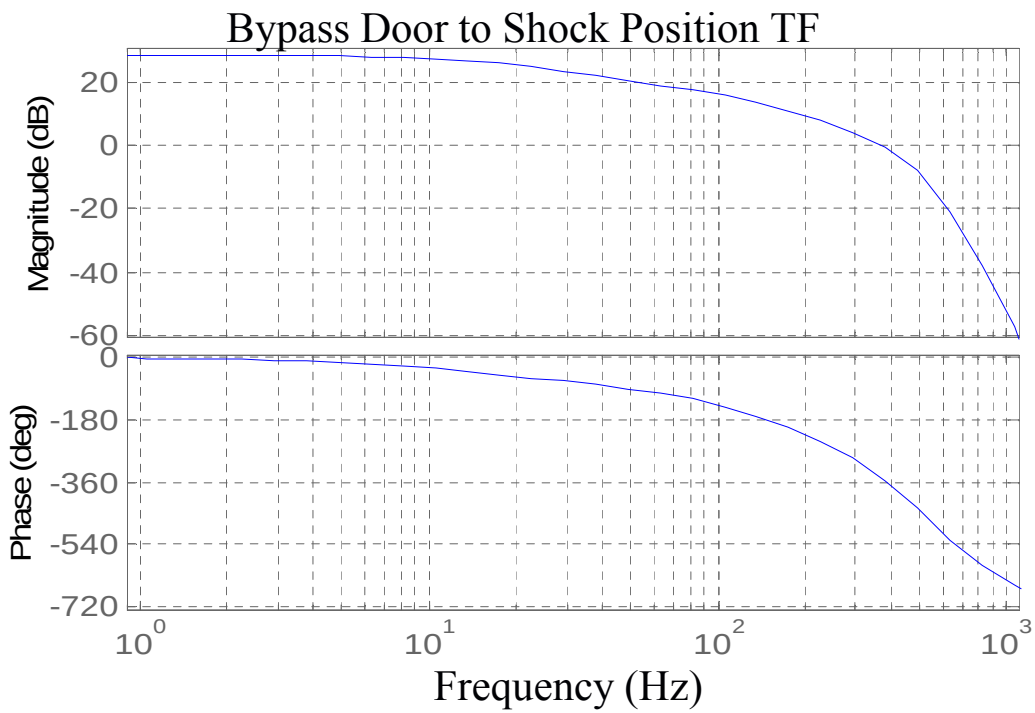
Shock Position Feedback Control Diagram

Inlet Shock Position Controls

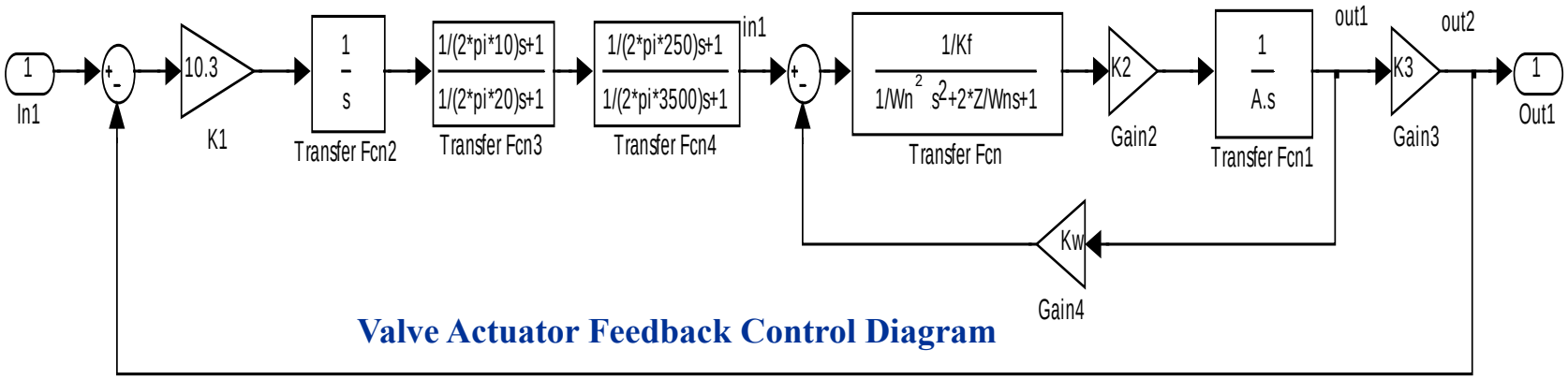
-- Plant TFs:

Bypass door to shock position (Using Large Perturbation Inlet (LAPIN) simulation

$$G_{BS}(s) = \frac{3.325K_{cr}(s/653+1)(s/2513+1)}{(s/118+1)(s^2/911^2+1.5s/911+1)(s^2/3142^2+1.1s/3142+1)(s^2/3330^2+1.3s/3330+1)} * \frac{1}{(s^2/3519^2+1.3s/3519+1)(s^2/3707^2+1.3s/3707+1)}$$

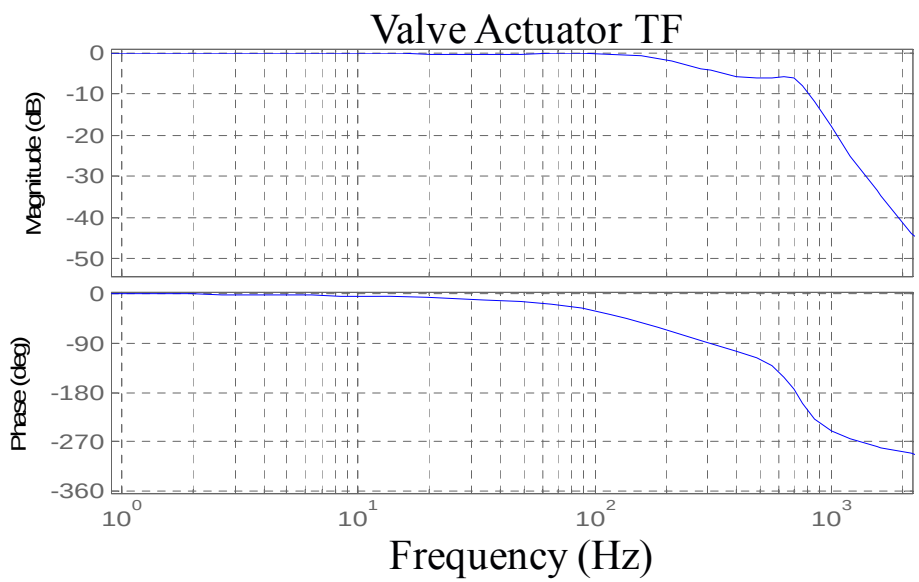


-- Plant TFs: Valve actuator to bypass door position



Valve Actuator Feedback Control Diagram

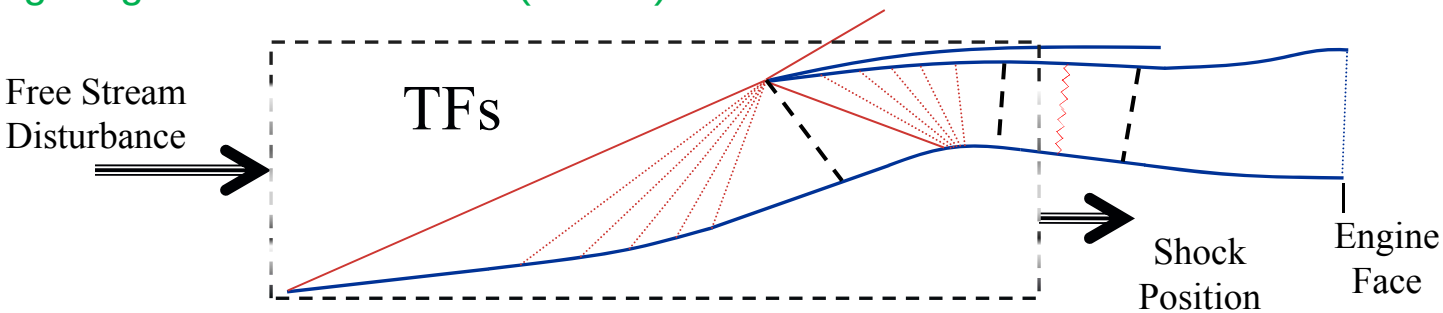
$$G_A(s) = \frac{(s/63+1)}{(s/60+1)(s^2/1136^2+0.8s/1136+1)(s^2/4753^2+0.6s/4753+1)}$$



Inlet Disturbance Transfer Functions

Freestream disturbances to shock Position

Using Large Perturbation Inlet (LAPIN) simulation



Pressure:

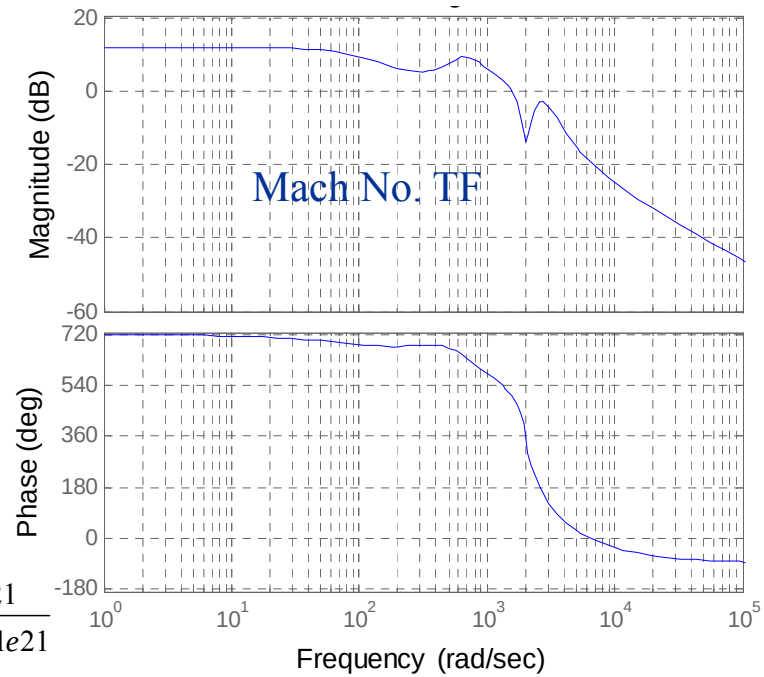
$$G_{PS}(s) = \frac{1.55s^5 - 2.08e4s^4 + 5.37e7s^3 - 8.19e10s^2 + 2.16e14s - 8.75e14}{s^6 + 3.09e3s^5 + 1.07e7s^4 + 1.78e10s^3 + 1.66e13s^2 + 8.62e15s + 6.86e17}$$

Temperature:

$$G_{TS}(s) = \frac{-4.89s^7 + 1.78e4s^6 - 5.69e7s^5 + 1.28e11s^4 - 2.20e14s^3 + 1.12e17s^2 - 2.93e20s - 6.26e22}{s^8 + 3.87e3s^7 + 1.51e7s^6 + 3.46e10s^5 + 5.69e13s^4 + 7.02e16s^3 + 4.97e19s^2 + 2.65e22s + 3.47e24}$$

Mach Number:

$$G_{MS}(s) = \frac{4.88e2s^6 - 3.24e6s^5 + 9.16e9s^4 - 1.01e13s^3 + 2.69e16s^2 + 1.52e19s + 4.69e21}{s^7 + 3.12e3s^6 + 1.42e7s^5 + 2.16e10s^4 + 3.88e13s^3 + 2.23e16s^2 + 1.31e19s + 1.21e21}$$



Atmospheric Turbulence

"Atmospheric Turbulence Modeling for Aero Vehicles – Fractional Order Fits," NASA/TM submitted for publication, George Kopasakis

Approach:

Taking an existing atmospheric wind gust model, also scaling it for temperature and pressure and deriving whole order TF's to fit the fractional order disturbances (for ε=8.6e-5, L=765)

Longitudinal acoustic velocity (m/sec):

$$G_{LA}(s) = \frac{8.74(s/9.2+1)(s/55.0+1)(s/335.5+1)}{(s/1.46+1)(s/30.1+1)(s/85.7+1)(s/1593.1+1)}$$

Temperature (°K):

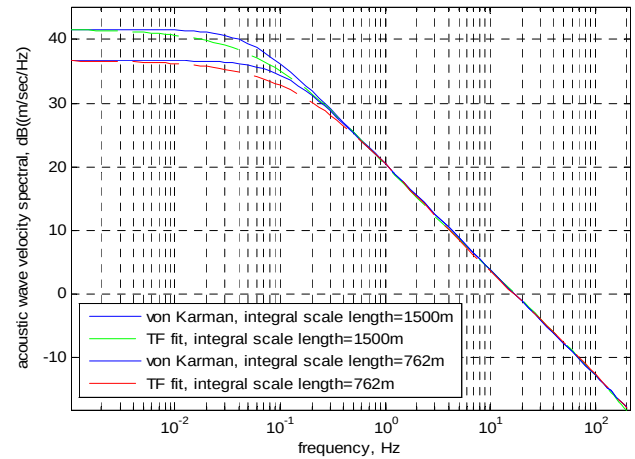
$$G_T(s) = \frac{41.75(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

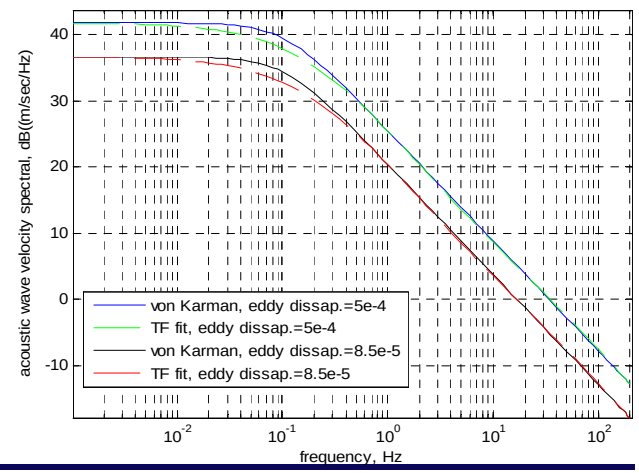
Acoustic Velocity due to Temperature (m/sec):

$$G_{TA}(s) = \frac{M\gamma R}{a_o}\frac{20.88(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

Pressure (Pa):

$$G_P(s) = \frac{37.96(s/33.0+1)(s/45.6+1)(s/602.4+1)}{(s/1.1+1)(s/25.1+1)(s/109.8+1)(s/816.3+1)}$$

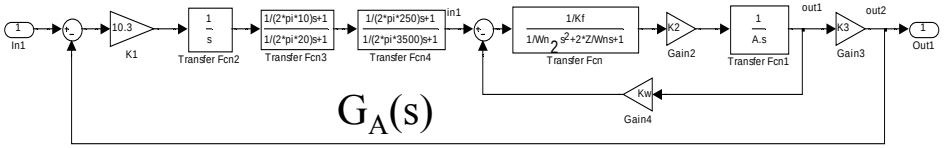


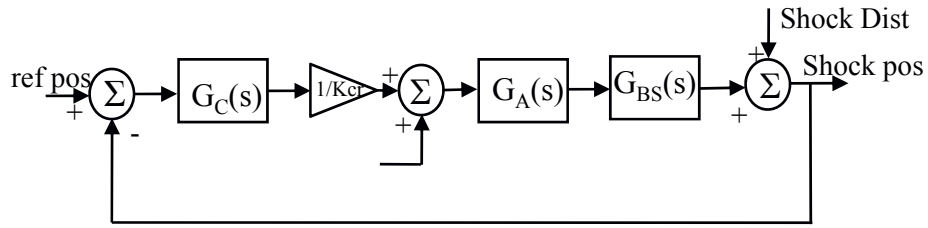


Inlet Shock Position Controls Design Process

Multi-Loop Controls Design Approach:

1. **For Max. Disturbance Attenuation, starting with a high shock controller bandwidth (145 Hz –vs.- 175 Hz bandwidth of the actuator loop)**





2. **Design shock CLG (steps a-d) to maximize disturbance attenuation w/ sufficient stability margins (20 db mid-frequency disturbance atten., $\phi M > 60\,\mathrm{deg.}$)**

3. **Lower proportional controller gain until control loops are sufficiently decoupled (bandwidth will proportionally decrease as well)**

4. **Repeat Step 2. w/ new shock controller bandwidth**

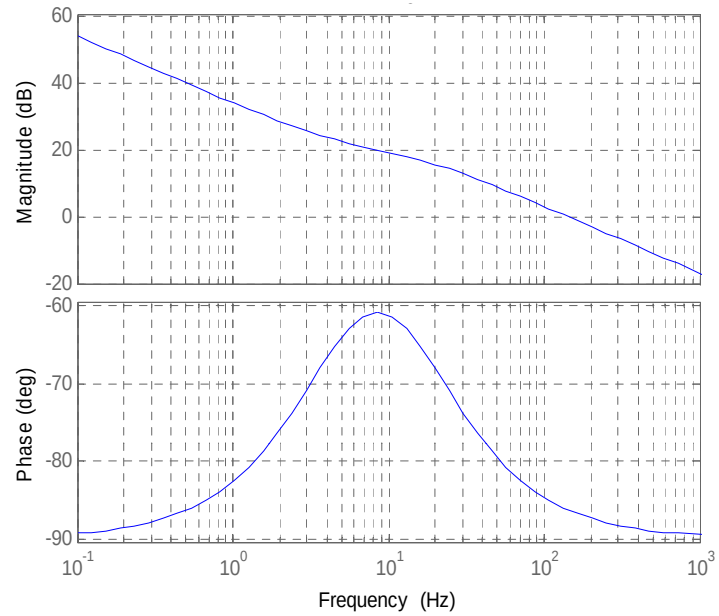
5. **If this doesn't meet design objectives (command tracking, margin reduction, etc) what's left is to ask for more hardware capability**

Kopasakis et al. – Shock Positioning Controls Design for a Supersonic Inlet, AIAA-2009-5117

Inlet Shock Position Controls Design Process

2. Designed shock controller closed loop gain (CLG):

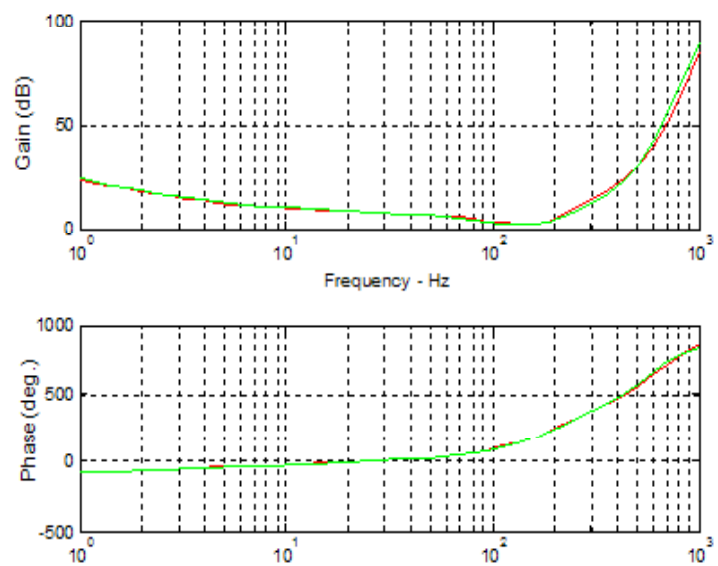
$$L(s) = 310\frac{(s/(2\pi 5)+1)}{s(s/(2\pi 14.5)+1)}$$



$$L(s) = G_C(s)G_A(s)G_{BS}(s)$$

$$|G_C(s)| = 20\log_{10}|L(s)| - 20\log_{10}|G_A(s)G_{BS}(s)|$$

$$\phi(G_C(s)) = \phi(L(s)) - \phi(G_A(s)G_{BS}(s))$$



Inlet Shock Position Controls Design Process

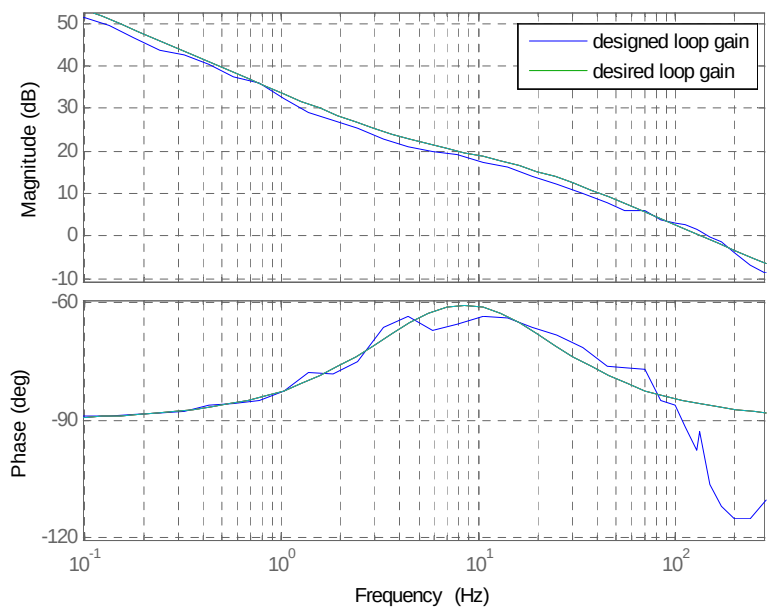
Shock Controller from pole/zero TF fit:

$$G_C(s) = \frac{310(s/2\pi 5+1)(s/2\pi 18.8+1)(s^2/(2\pi 145)^2+0.98s/2\pi 145+1)(s^2/(2\pi 250)^2+1.3s/2\pi 250+1)}{s(s/2\pi 14.5+1)(s/2\pi 104+1)(s^2/(2\pi 5000)^2+1.4s/2\pi 5000+1)(s^2/(2\pi 5500)^2+1.4s/2\pi 5500+1)}$$

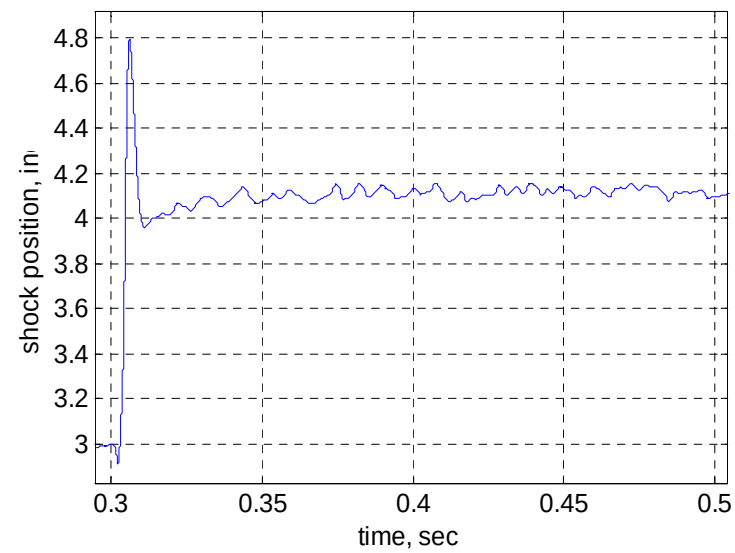
$$\frac{(s^2/(2\pi 350)^2+1.36s/2\pi 350+1)(s^2/(2\pi 450)^2+1.0s/2\pi 450+1)}{(s^2/(2\pi 6000)^2+1.4s/2\pi 6000+1)(s^2/(2\pi 6500)^2+1.4s/2\pi 6500+1)}$$

$$\frac{(s^2/(2\pi 550)^2+1.0s/2\pi 550+1)(s^2/(2\pi 650)^2+0.60s/2\pi 650+1)}{(s^2/(2\pi 7000)^2+1.4s/2\pi 7000+1)(s^2/(2\pi 7500)^2+1.4s/2\pi 7500+1)}$$

Verify CLG design



Step Response



Inlet Shock Position Controls Design Process

3. Lower controller proportional gain until control loops are decoupled

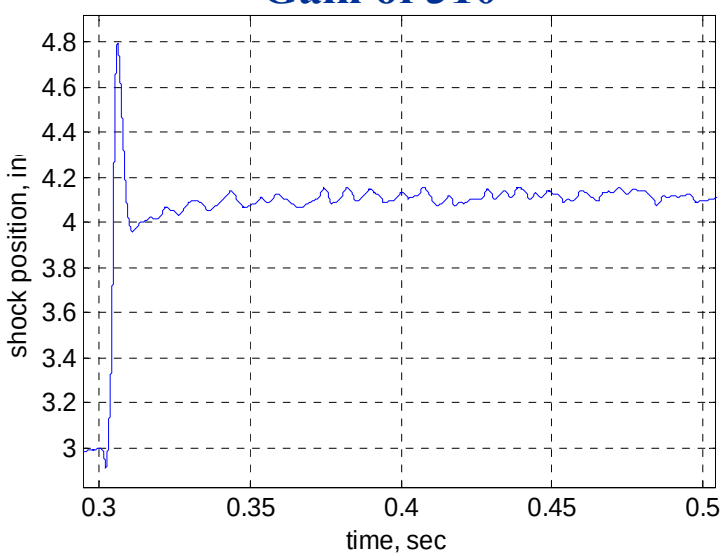
-- Sufficient decoupling when proportional gain lower to 100 from 310

-- Mid-frequency disturbance attenuation will drop to ~10 dB

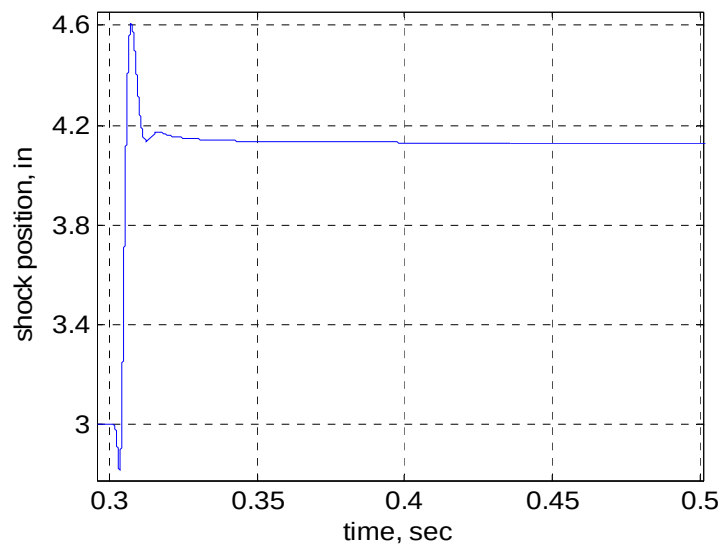
-- Shock position control bandwidth falls to 35 Hz

Step Responses w/ gain of 100

Gain of 310



Gain of 100

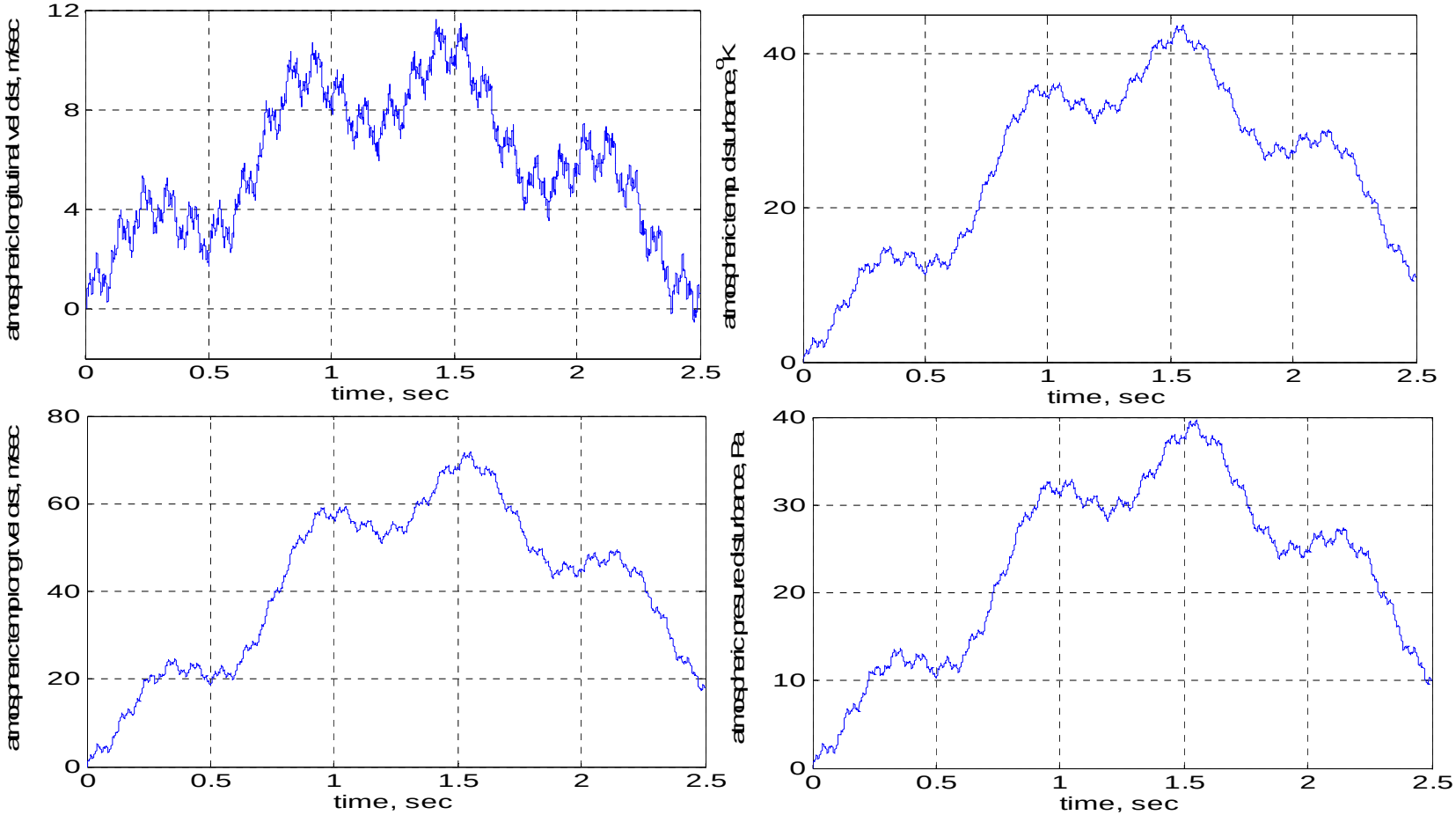


-- Check shock control system performance with reduced gain

Atmospheric Turbulence

Worst case turbulence at over 20k feet altitude

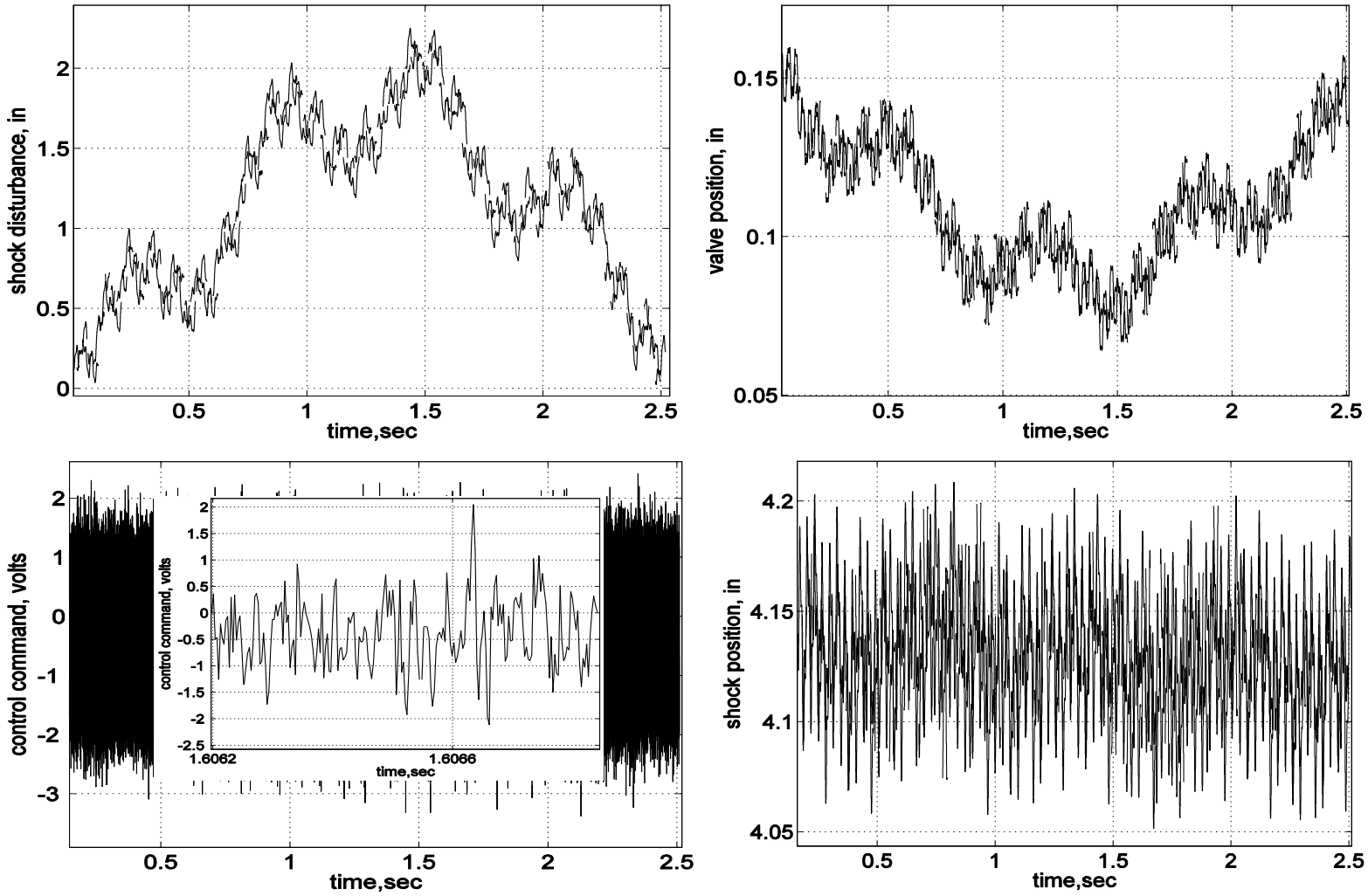
-- combined longitudinal acoustic velocity gust w/ temperature fluctuation (temp + acoustic velocity due to temp), w/ pressure fluctuation



Shock Control System Command Tracking Performance

-- Control parameters within bounds

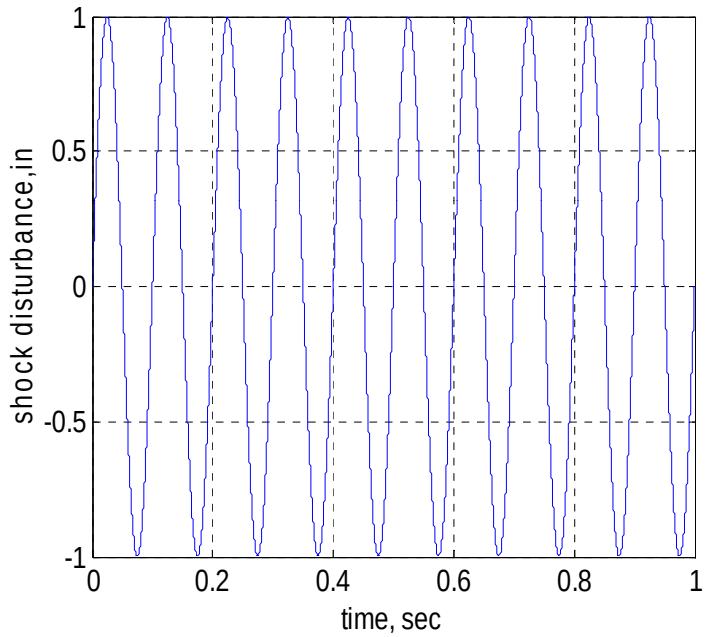
-- Shock position tracking within approx. 0.15 in

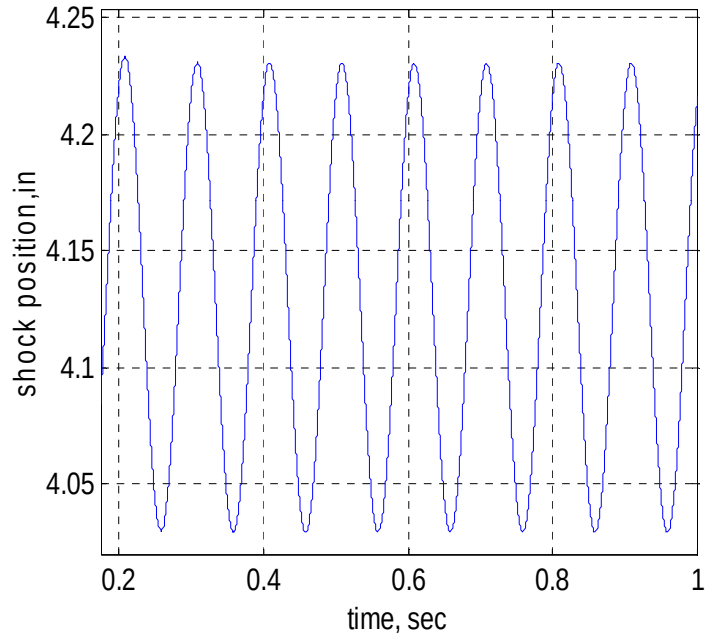


Shock Control System Command Tracking Performance

Check Mid-frequency disturbance attenuation

Applying 10 HZ disturbance at the shock position

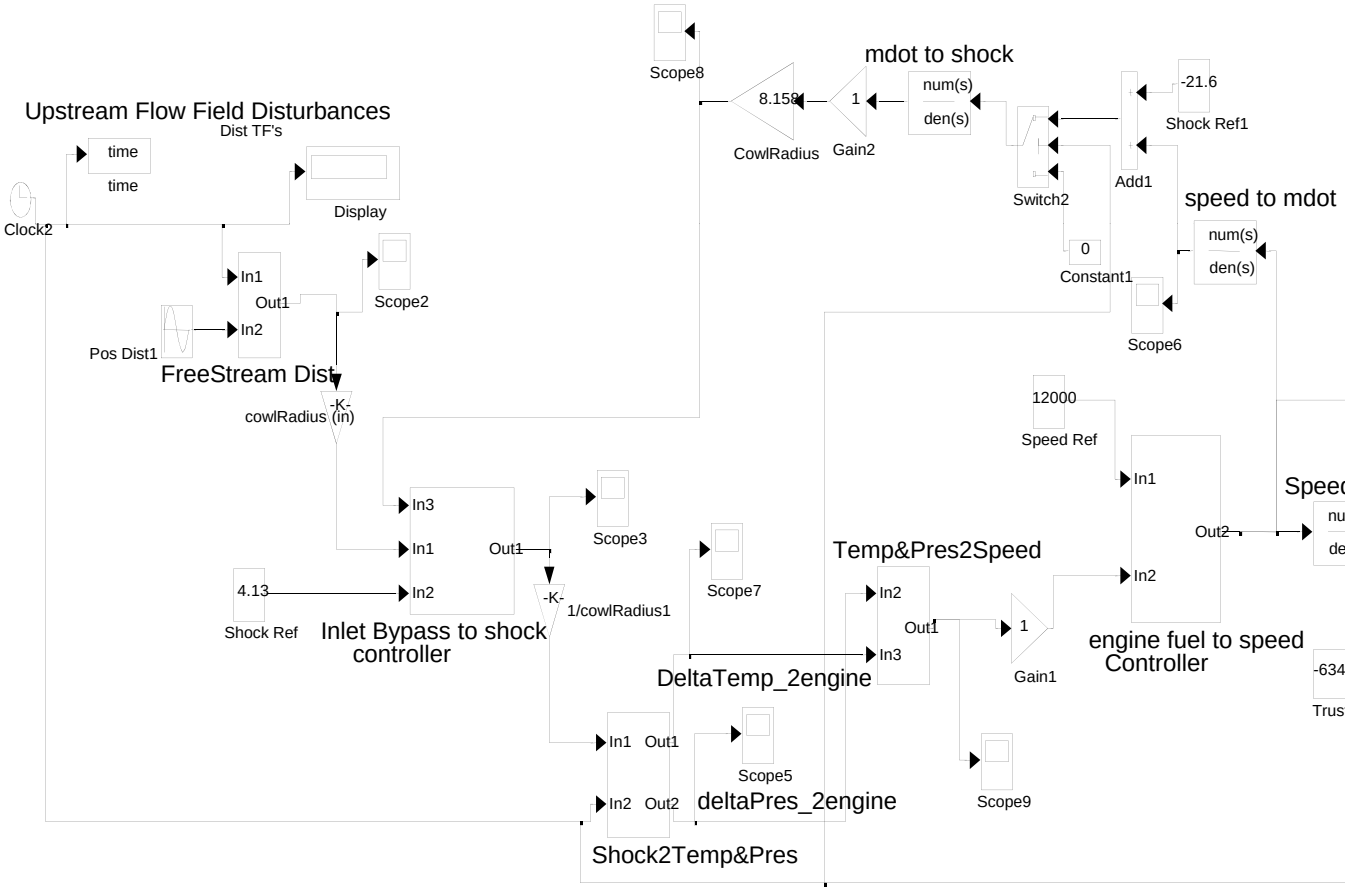


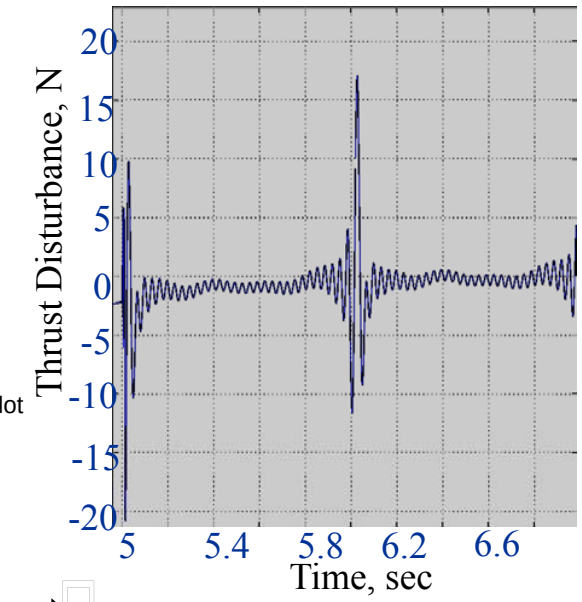


-- Mid Frequency attenuation of ~ 20 dB instead of 10 db due to lower gain (aided by the actuator control loop)

Integrated Linear Propulsion System Model

-- Integrated linear propulsion system model developed





Conclusion

- ☆ **Shock controls design approach developed to maximize command tracking performance in the presence of disturbance**
- ☆ **More work needed to include other disturbance, like pitch, yaw, and aeroservoelastic**
- ☆ **Additional work needed to study integrated propulsion and aeropropulsoservoelastic effects**
- ☆ **So far results are encouraging;**
  If tight shock position control can be maintained, reduction in shock control margin and significant propulsion efficiency gains can be realized as well as reduction in thrust transients