Onboard Model-Based Aircraft Engine Performance Estimation

Donald L. Simon
Sanjay Garg
NASA Glenn Research Center

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Model-Based Aircraft Engine Performance Estimation

Outline

- Problem Statement
- Background
- Approach
- Results
- Conclusions
- Future Plans
Model-Based Aircraft Engine Performance Estimation

Problem Statement

• The accurate estimation of unmeasured aircraft gas turbine engine performance parameters is an enabling technology area for:
  – Detection
  – Diagnostics
  – Prognostics
  – Controls

• Challenges:
  – Each engine will exhibit a unique level of performance due to deterioration and manufacturing tolerances
  – Poses an underdetermined estimation problem (i.e., more unknowns than available sensor measurements).

• Current state of the art:
  – Estimates are based upon fleet-average engine models
  – Emerging approach: onboard adaptive model-based estimation

• Goal: Develop a methodology for minimizing the onboard model-based estimation error when facing the underdetermined estimation problem.
**Background**

**Adaptive Onboard Model-Based Performance Estimation**

**Background:**
- Adaptive on-board engine model embedded within engine control computer
- A tracking filter automatically tunes the onboard model to match the physical engine performance
- The tracking filter is typically based upon Kalman filter estimation concepts

**Challenges:**
- Performance deterioration and underdetermined estimation problem can cause errors between estimated and actual engine performance.
Objective

Objective: develop a systematic methodology for combined sensor and model tuning parameter selection that minimizes the linear Kalman filter estimation error.
Problem Formulation
Linearized Engine Model

State equations: \[ x_{k+1} = Ax_k + Bu_k + Lh_k + w_k \]

Output equations: \[ y_k = Cx_k + Du_k + Mh_k + v_k \]

Auxiliary equations: \[ z_k = Fx_k + Gu_k + Nh_k \]

- \( x_k \): state variables (spool speeds)
- \( y_k \): measured variables (spool speeds, temperatures, pressures)
- \( z_k \): auxiliary outputs (thrust, stall margins)
- \( u_k \): control inputs (fuel flow, variable stator vanes, variable bleed)
- \( h_k \): engine health parameters (component efficiency, flow capacity)
- \( w_k \): process noise (zero mean, normally distributed)
- \( v_k \): sensor noise (zero mean, normally distributed)

\( z \) can be computed if \( x, u, \) and \( h \) are known
Problem Formulation

Linearized Engine Model

State equations: \[ x_{k+1} = Ax_k + Bu_k + Lh_k + w_k \]

Output equations: \[ y_k = Cx_k + Du_k + Mh_k + v_k \]

Auxiliary equations: \[ z_k = Fx_k + Gu_k + Nh_k \]

Engine performance deterioration evolves slowly, thus health parameters are typically modeled without dynamics \[ h_{k+1} = h_k \]
Problem Formulation

Linearized Engine Model

State equations:

\[
\begin{bmatrix}
    x_{k+1} \\
    h_{k+1}
\end{bmatrix}
= \begin{bmatrix}
    A & L \\
    0 & I
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    h_k
\end{bmatrix}
+ \begin{bmatrix}
    B \\
    0
\end{bmatrix}
\begin{bmatrix}
    u_k
\end{bmatrix}
+ \begin{bmatrix}
    w_k \\
    w_{h,k}
\end{bmatrix}
\]

Output equations:

\[
y_k = \begin{bmatrix}
    C & M \\
    C_{xh}
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    h_k \\
    x_{xh,k}
\end{bmatrix}
+ D u_k + v_k
\]

Auxiliary equations:

\[
z_k = \begin{bmatrix}
    F & N \\
    F_{xh}
\end{bmatrix}
\begin{bmatrix}
    x_k \\
    h_k \\
    x_{xh,k}
\end{bmatrix}
+ G u_k
\]

Once \( h \) is part of the augmented state vector, it can be estimated using a Kalman filter as long as the system is observable.

It is a necessary condition for observability that there be at least as many sensors as health parameters to be estimated.

Conventional approach is to only estimate a subset of health parameters and assume others remain constant.
Reduced-order state space equations are constructed by defining a reduced-order tuner vector, \( q \), that is a linear combination of all health parameters, and of appropriate dimension to enable Kalman estimation:

\[
q = V^* h
\]

- \( q \) : reduced order model tuner vector
- \( h \) : engine health parameters
- \( V^* \) : transformation matrix

Formulation of the reduced-order state space equations:
- \( h \) is replaced with \( q \)
- \( L, M, \) and \( N \) matrices are post-multiplied by \( V^* \)
Problem Formulation
Kalman Filter Formulation

In this study steady-state Kalman filtering is applied

At steady-state open loop operating conditions \((u = 0)\), the Kalman filter estimator is given as

\[
\hat{x}_{xq,k} = \begin{bmatrix} \hat{x}_k \\ \hat{q}_k \end{bmatrix} = A_{xq} \hat{x}_{xq,k-1} + K_\infty \left( y_k - C_{xq} A_{xq} \hat{x}_{xq,k-1} \right)
\]

The estimate of \(x_{xh,k}\) can be converted into estimates of \(h_k\) and \(z_k\)

\[
\hat{x}_{xh,k} = \begin{bmatrix} \hat{x}_k \\ \hat{h}_k \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & V^{*\dagger} \end{bmatrix} \hat{x}_{xq,k}
\]

\[
\hat{z}_k = \begin{bmatrix} F & NV^{*\dagger} \end{bmatrix} \hat{x}_{xq,k}
\]
Analytical Derivation of Kalman Estimation Error

Squared estimation error bias

\[ \text{SSEE}(\hat{z}_{\text{bias}}) = \text{trace}\left\{ E\left[ \hat{z}_k - z_k \right] (E\left[ \hat{z}_k - z_k \right])^T \right\} \]

\[ = \text{trace}\left\{ G_z P_h G_z^T \right\} \]

where

\[ G_z = \begin{bmatrix} F & NV^{*+} \\ & (I - A_{xq} + K_{xq} C_{xq} A_{xq})^{-1} \end{bmatrix} \ldots \]

\[ \times K_{xq} \begin{bmatrix} C(I - A)^{-1} L + M \end{bmatrix} \ldots \]

\[ - \begin{bmatrix} F(I - A)^{-1} L + N \end{bmatrix} \]

and \( P_h \) reflects a priori knowledge of the covariance in health parameters

Estimation variance

\[ \text{SSEE}(\hat{z}_{\text{var}}) = \text{trace}\left\{ E\left( \hat{z}_k - E[\hat{z}_k] \right) (\hat{z}_k - E[\hat{z}_k])^T \right\} \]

\[ = \text{trace}\left\{ F \begin{bmatrix} NV^{*+} \end{bmatrix} P_{\hat{x},k} \begin{bmatrix} F & NV^{*+} \end{bmatrix} \right\} \]

where \( P_{\hat{x},k} \) is obtained by solving the following Ricatti equation

\[ P_{\hat{x},k} = \begin{bmatrix} A_{xq} & -K_{xq} C_{xq} A_{xq} \end{bmatrix} P_{\hat{x},k} \begin{bmatrix} A_{xq} & -K_{xq} C_{xq} A_{xq} \end{bmatrix} \ldots \]

\[ + K_{xq} R K_{xq}^T \]

Squared bias and variance are combined to form sum of squared estimation errors (SSEE)

\[ \text{SSEE}(\hat{z}_{\text{fleet}}) = \text{trace}\left\{ G_z P_h G_z^T + F \begin{bmatrix} NV^{*+} \end{bmatrix} P_{\hat{x},k} \begin{bmatrix} F & NV^{*+} \end{bmatrix} \right\} \]

Optimization: Choose \( V^{*} \) and sensors to minimize SSEE in the parameters of interest
1. Apply an exhaustive search considering all possible sensor combinations.
2. For each candidate sensor suite apply a Matlab-based optimal iterative search to determine optimal $V^*$
3. Choose sensor/tuner combination that minimizes SSEE or WSSEE

$V^*$ optimal iterative search flow chart
Model-Based Aircraft Engine Performance Estimation

Turbofan Engine Example

Applied to NASA Commercial Modular Aero-Propulsion System Simulation (C-MAPSS)

State equations: \( x_{k+1} = Ax_k + Bu_k + Lh_k + w_k \)

Output equations: \( y_k = Cx_k + Du_k + Mh_k + v_k \)

Auxiliary equations: \( z_k = Fx_k + Gu_k + Nh_k \)

<table>
<thead>
<tr>
<th>State Variables ( x )</th>
<th>Health Parameters ( h )</th>
<th>Actuators ( u )</th>
<th>Sensors ( y )</th>
<th>Auxiliary Outputs ( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nf</td>
<td>WF36</td>
<td>Nf</td>
<td>T40</td>
</tr>
<tr>
<td>2</td>
<td>Nc</td>
<td>VBV</td>
<td>Nc</td>
<td>T50</td>
</tr>
<tr>
<td>3</td>
<td>LPC efficiency</td>
<td>VSV</td>
<td>P24</td>
<td>Fn</td>
</tr>
<tr>
<td>4</td>
<td>LPC flow capacity</td>
<td>T24</td>
<td></td>
<td>SmLPC</td>
</tr>
<tr>
<td>5</td>
<td>HPC efficiency</td>
<td>Ps30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>HPC flow capacity</td>
<td>T30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>HPT efficiency</td>
<td>T48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>HPT flow capacity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>LPT efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>LPT flow capacity</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Model-Based Aircraft Engine Performance Estimation

Turbofan Engine Example (cont.)

• Designed different Kalman filters applying the following two approaches:
  – Tuners selected as a subset of health parameters (conventional approach) – exhaustive search applied to find the best “subset”
  – Systematic tuner selection (approach developed under IVHM project)

• Theoretically derived the Kalman filter mean squared estimation error for each Kalman filter

• Conducted Monte Carlo simulation studies to experimentally validate estimation errors
  – Health parameter vectors are normally distributed in accordance with covariance matrix, $P_h$
  – Sensor noise included
  – Ran 375 test cases, each 30 seconds in duration, $\Delta t = 15$ ms
Model-Based Aircraft Engine Performance Estimation

Monte Carlo Simulation (z estimation results)

### Auxiliary parameter mean squared estimation errors

<table>
<thead>
<tr>
<th>Tuners</th>
<th>Error</th>
<th>T40 (°R)</th>
<th>T50 (°R)</th>
<th>Fn (%)</th>
<th>SmLPC (%)</th>
<th>WSSEE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Subset of health parameters</strong></td>
<td>Theor. sqr. bias</td>
<td>0.00</td>
<td>191.35</td>
<td>1.33</td>
<td>2.47</td>
<td></td>
</tr>
<tr>
<td>(conventional approach)</td>
<td>Theor. variance</td>
<td>73.56</td>
<td>26.37</td>
<td>0.29</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Theor. sqr. error</td>
<td>73.56</td>
<td>217.72</td>
<td>1.62</td>
<td>2.79</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>Exper. sqr. error</td>
<td>73.97</td>
<td>215.41</td>
<td>1.65</td>
<td>2.85</td>
<td>0.53</td>
</tr>
<tr>
<td><strong>Systematic tuner selection</strong></td>
<td>Theor. sqr. bias</td>
<td>0.00</td>
<td>84.95</td>
<td>0.66</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>(new approach)</td>
<td>Theor. variance</td>
<td>24.19</td>
<td>20.58</td>
<td>0.14</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Theor. sqr. error</td>
<td>24.19</td>
<td>105.53</td>
<td>0.80</td>
<td>1.47</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Exper. sqr. error</td>
<td>24.34</td>
<td>101.44</td>
<td>0.78</td>
<td>1.41</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Average mean squared estimation error reduction (experimental) = 53%
Average mean estimation error reduction (experimental) = 32%
Thrust estimation accuracy comparison of conventional vs. enhanced estimation approaches

Conventional estimation approach

![Graph showing noticeable error between estimated and actual thrust](image)

Enhanced estimation approach

![Graph showing minimal error between estimated and actual thrust](image)
Integrated tuner and sensor selection was also performed

- Also applied to C-MAPSS Turbofan Engine Model
- Objective: Minimize health parameter ($h$) estimation error
- Assumed a baseline sensor suite of five sensors
  - Nf, Nc, T24, Ps30, T48
- Considered six optional sensors
  - P24, T30, P45, P50, T50, P15
- Selected optimal tuner and sensor suite for suites of 5 to 11 sensors in size
- Selection was performed applying an exhaustive search (considered all tuner and sensor combinations for a given target number of sensors)
### Integrated Tuner and Sensor Selection

#### Sensor Selection Legend:
- **X** = Conventional tuner selection approach
- **O** = Enhanced tuner selection approach

#### Findings:
- Estimation accuracy improves as additional sensors are added
- The enhanced tuner selection approach provides superior estimation accuracy relative to the conventional approach
- Optimal sensor suite is dependent on the tuner selection approach applied

#### Table: Health Parameter Mean Sum of Squared Estimation Errors (SSEE)

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Health Parameter Mean Sum of Squared Estimation Errors (SSEE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional Approach ((q = \text{subset of } h))</td>
</tr>
<tr>
<td>5</td>
<td>35.40</td>
</tr>
<tr>
<td>6</td>
<td><strong>26.01</strong></td>
</tr>
<tr>
<td>7</td>
<td><strong>20.51</strong></td>
</tr>
<tr>
<td>8</td>
<td>15.28</td>
</tr>
<tr>
<td>9</td>
<td>10.90</td>
</tr>
<tr>
<td>10</td>
<td><strong>7.77</strong></td>
</tr>
<tr>
<td>11</td>
<td><strong>7.11</strong></td>
</tr>
</tbody>
</table>

#### Graph: Sum of Squared Estimation Errors (SSEE) vs. Number of Sensors

- **q = \text{subset of } h (conventional approach)**
- **q = \text{combination of all elements of } h (enhanced approach)**
Conclusions

• Sensor and model tuning parameter selection decisions have a significant impact on onboard Kalman filter estimation accuracy.

• The optimal sensor and model tuner vector combination will vary dependent upon the estimation objective.

• The presented methodology provides designers a systematic approach for performing combined sensor and tuner selection for their individual applications.

• Methodology shown to yield a significant improvement in Kalman filter estimation accuracy.
The systematic tuner/sensor selection methodology will be used to support future propulsion gas path diagnostics research under the IVHM Project. Areas for future work include:

- Extending the technique to select tuning parameters optimal over a range of operating conditions
- Applying piecewise linear (full envelope) Kalman filter estimation
- Applying the technique to support future IVHM Project milestones in performance estimation and onboard diagnostics.
Conference papers/presentations:


Journal article: