THE INFLUENCE OF ELASTIC DEFORMATION UPON THE MOTION OF A BALL ROLLING BETWEEN TWO SURFACES

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When a ball rolls between two surfaces, in general, a tangential contact force and a relative angular velocity of spin are present at each point of contact. Both these actions give rise to tangential frictional tractions transmitted across the contact surface which are shown to result in a velocity of creep of the ball in a direction perpendicular to the nominal rolling path.

The magnitude of the creep velocity depends critically upon the magnitudes of the tangential force and the velocity of spin. If these actions are small there is negligible slip between the contacting surfaces and the creep motion is predominantly a function of the elastic properties of the materials. At larger spin velocities slip extends over a greater proportion of the contact area and the creep is influenced by the frictional properties of the surfaces.

Creep measurements have been made over a wide range of conditions of rolling. The results are reduced to non-dimensional form, in terms of two parameters expressing the effect of tangential forces and spin respectively.

The resistance to rolling has been measured and is shown to control the axis about which the ball rolls. The detailed mechanism of the rolling process is discussed.

INTRODUCTION

When a sphere is used as a rolling element between two surfaces it is usual to assume that the relative motion of sphere and the rolling surfaces is not influenced in any significant way by their elastic deformation. To take the rolling bodies as rigid implies 'point-contact' between them, whereupon their relative motion is exactly determined by the simple geometry and constraints of the particular arrangement. Of course contact does not occur at a point. The force, normal to the surfaces, transmitted by the rolling element causes local distortion, albeit elastic, so that contact occurs over a finite area of elliptical shape defined by the Hertz theory. Owing to friction, tangential tractions may be transmitted between the rolling surfaces across this area of contact which give rise to a relative velocity between the two surfaces, which is small compared with the rigid-body motion, usually referred to as 'creep'.

The creep motion results from a small amount of slip between the surfaces caused by their elastic distortion. It has been shown that *slip* may and, in fact, does take place over part of the area of contact, even though the two bodies may be rolling together without *sliding*†. It is the purpose of this paper to demonstrate the nature of the creep motion and how the track of a ball rolling between two surfaces is affected by it.

The influence of elasticity upon the motion of a sphere rolling on a single plane surface has been studied by the author previously (1)‡. Here the motion was classified into three categories, namely,

(1) *Free rolling*, in which the force transmitted between the sphere and the plane acts normal to the contact surface, and further there is no angular velocity of 'spin' between the two.

(2) *Rolling with tangential forces* which takes place when the transmitted force has a tangential component less than the limiting friction force.

(3) *Rolling with spin* which occurs when the two bodies have a relative angular velocity about an axis normal to the contact surface.

A convenient arrangement for studying the problem of a sphere rolling between two surfaces is shown in Fig. 2.

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† *The term* slip *is used here to describe relative velocity between the surfaces at a particular point, or points, within the contact area, whilst* slide *refers to a complete bodily movement in which slipping is taking place at all points in the contact area.*

‡ *A numerical list of references is given in the Appendix.*

The lower surface is fixed whilst the upper surface rotates about the axis Q–Q. It is evident that, neglecting the creep motion, the ball will roll along a curved path of radius R. The case of rolling along a straight path is then the limiting one, when the axis of rotation is an infinite distance away ($R \to \infty$). For the rolling conditions to remain steady the two surfaces must be surfaces of revolution about the axis Q–Q, inclined to each other at the points of contact by angle 2β. Rolling between parallel surfaces is then the limiting case where $\beta = 0$.

The conditions of rolling at each point of contact may be analysed into the categories mentioned above. It is well known that conditions of free rolling—sometimes referred to as 'pure' rolling—cannot obtain concurrently at both points of contact (except in the limiting case of straight rolling between parallel surfaces), and are unlikely to occur at either. An angular velocity of spin between the ball and the discs is normally present. In addition, tangential contact forces perpendicular to the direction of rolling may arise, either from the inclination of the two surfaces at the contact points, or from body forces due to centrifugal or gyroscopic action. In general, then, both a transverse tangential force and a motion of spin are present at each point of contact.

A tangential contact force gives rise to a creeping motion of the sphere relative to the plane on which it rolls in a direction directly opposite to that of the force transmitted to the sphere. A transverse tangential force, therefore, produces a creep velocity at right angles to the direction of rolling. The action of spin has been shown theoretically and confirmed experimentally (1b) also to give rise to a transverse creep. Theoretical relationships have been derived for the creep velocity in terms of (*a*) the applied tangential force and (*b*) the angular velocity of spin. These formulae might be expected to apply when the tangential force and the spin velocity are both *small*. Physically, this condition corresponds to the restriction that slip should be taking place to a negligible extent throughout the contact area*.

For larger values of the spin velocity, when slip is taking place to an appreciable extent, it has not yet been possible to solve the mixed boundary value problem in the theory of elasticity to obtain a theoretical expression for the creep velocity. This range of the creep problem has been studied by experiment only.

THEORETICAL

Creep Motion of a Ball Rolling on Plane

In studying the kinematics of rolling motion it is convenient to express the velocities relative to a stationary point (or, more exactly, area) of contact. The origin, O, of rectangular co-ordinate axes is taken at the centre of the contact area. Axes Ox and Oy lie in the undistorted plane of rolling, with Ox in the direction of rolling (Fig. 1). In this view a steady rolling motion appears as a stationary pattern of elastic distortion in the contact region through which the material of both bodies flows at a steady rate.

The ball is pressed on to the plane by a normal force N which produces contact over a circular area of radius a, given by Hertz:

$$a^3 = \frac{3(1-\nu)Nr}{4G} \quad . \quad . \quad . \quad (1)$$

where r is the radius of the ball; G is the modulus of rigidity and ν is Poisson's ratio for the material of both surfaces.

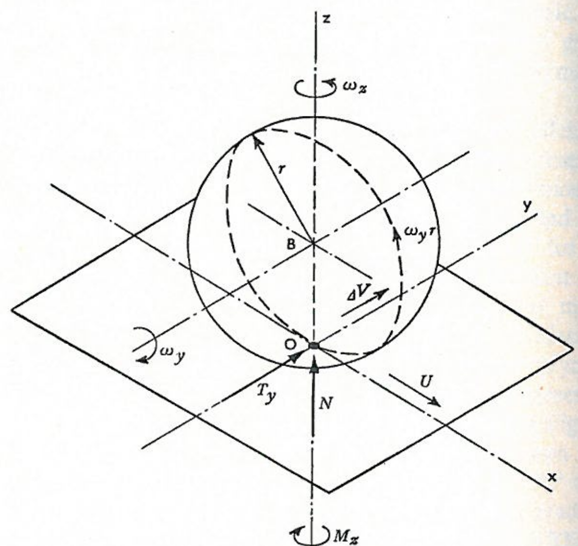


Fig. 1. The Co-ordinate System for a Sphere Rolling on a Plane

The steady rolling velocity, parallel to Ox, is denoted by U. In the absence of a tangential force in the direction of rolling ($T_x = 0$) the ball will have an angular velocity ω_y about an axis parallel to Oy, given by:

$$U = \omega_y r \quad . \quad . \quad . \quad . \quad (2)$$

A transverse tangential force $T_y (< \mu N)$ is transmitted from the plane to the sphere which gives rise to a transverse velocity of creep of the sphere relative to the plane denoted by ΔV. The creep ratio is defined by: $\xi_y = \Delta V / U$. An angular velocity of spin of the sphere relative to the plane about the normal axis Oz is denoted by ω_z.

Formulae have been obtained for the transverse creep ratio due to the action of a tangential force and a spin velocity taken separately ((1a) equation (46) and (1b) equation (26)). These equations are linear relationships; they have been derived on the assumption of perfectly elastic solids with no interfacial slip. It is, therefore, justifiable to superpose the results to cover the case when

* *It has been shown* (loc. cit.) *that slip initiates at the 'trailing edge' of the area of contact under the action of the smallest tangential force or spin motion, but, provided these actions are small, the creep velocity may be calculated on the assumption that the surfaces remain locked together, over the whole of the contact area.*

tangential forces and spin are present concurrently. Direct superposition of the results referred to above, making use also of equation (1), gives rise to an expression for the overall transverse creep:

$$\xi_y \frac{r}{a} = \frac{2(2-\nu)}{3(3-2\nu)}\left(\frac{\omega_z r}{U}\right) - \frac{(4-\nu)}{12(1-\nu)}\left(\frac{T_y}{N}\right) \quad . \quad (3)$$

This result applies provided that $(\omega_z r/U)$ and (T_y/N) are both small compared with μ, the coefficient of limiting friction between the two surfaces. Slip is then confined to a vanishingly thin 'new moon' at the trailing edge of the circle of contact.

The normal force N gives rise to the Herzian distribution of normal traction Z in the contact area, that is,

$$Z = Z_0 \frac{1}{a}\sqrt{a^2-x^2-y^2}, \quad (x^2+y^2) \leqslant a^2 \qquad (4)$$

where $Z_0 = 3N/2\pi a^2 =$ maximum pressure at $x = y = 0$. The tangential tractions X and Y due to the tangential force T_y and spin velocity ω_z are given in the references quoted ((1a) equation (49) and (1b) equations (23) and (24)). When the tangential force and spin act together the resultant tangential tractions over the area of contact $(x^2+y^2) \geqslant a^2$ become:

$$X/Z_0 = \frac{2(3-\nu)(1-\nu)}{3(3-2\nu)}\cdot\left(\frac{\omega_z r}{U}\right)\cdot\frac{(a+x)y}{a(a^2-x^2-y^2)^{\frac{1}{2}}} \quad . \quad (5)$$

and

$$Y/Z_0 = \frac{2(1-\nu)^2}{3(3-2\nu)}\cdot\left(\frac{\omega_z r}{U}\right)\cdot\frac{a^2-2x^2-ax-y^2}{a(a^2-x^2-y^2)^{\frac{1}{2}}} + \frac{1}{3}\left(\frac{T_y}{N}\right)\frac{a+x}{(a^2-x^2-y^2)^{\frac{1}{2}}} \quad . \quad (6)$$

It will be noted that the distributions of tangential traction given by equations (5) and (6) rise to infinity at the boundary of the contact circle $(x^2+y^2 = a^2)$, except at the leading point $(-a, 0)$ where they are both of zero value. Since the normal traction Z is zero at all points on this boundary we should expect slip to occur, thereby relieving the infinite tangential traction until its value everywhere falls to be equal to, or less than, the product $\mu \times Z$. However, provided that the values of $\omega_z r/U$ and T_y/N are both small compared with the value of μ, the extent of slip will be small and restricted to a thin band at the trailing boundary of the contact circle. The effect of slip on equations (2) and (3) should in this event be small.

It is relevant to note that the tangential tractions of equations (5) and (6) integrate to a resultant moment about the z-axis given by:

$$\frac{M_z}{Na} = -\frac{8(2-\nu)(1-\nu)}{3(3-2\nu)}\left(\frac{\omega_z r}{U}\right) + \frac{1}{3}\left(\frac{T_y}{N}\right) \qquad (7)$$

An important practical case of this problem occurs when the physical conditions contrive to eliminate the transverse creep motion. The left-hand side of equation (3) then becomes zero, and a fixed relationship must exist between the spin velocity and the tangential force, namely,

$$\left(\frac{T_y}{N}\right)_{\xi=0} = \frac{8(2-\nu)(1-\nu)}{(3-2\nu)(4-\nu)}\left(\frac{\omega_z r}{U}\right) \quad . \quad (8)$$

Substituting in equation (6) gives the net distribution of traction corresponding to this situation, and in equation (7) gives the moment about the z-axis,

$$\left(\frac{M_z}{Na}\right)_{\xi=0} = -\frac{8(2-\nu)(1-\nu)(3-\nu)}{3(3-2\nu)\,(4-\nu)}\left(\frac{\omega_z r}{U}\right) \qquad (9)$$

This moment resists the spin rotation and hence external work must be done to maintain the motion. The energy so supplied is dissipated by friction in the thin region of slip*.

Kinematics of a Ball Rolling Between Similar Concentric discs

The kinematic aspects of the problem of a ball used as a rolling element between two equal concentric discs are illustrated in Fig. 2a. The discs both rotate about the fixed axis Q–Q with angular velocities Ω_1 and Ω_2 whose values are such that the two points of contact O_1 and O_2 and the ball centre B remain stationary in space (leaving aside for a moment the creep velocities which are relatively small). The contact surfaces of the discs are taken to be plane, or of sufficiently small curvature for the contact area to be approximately circular, and are inclined to each other at angle 2β.

If the ball rolls without sliding at O_1 and O_2, the axis of rotation of the ball must lie in the radial plane (the plane of Fig. 2). The net angular velocity about this axis, inclined at an arbitrary angle ψ to the axis of symmetry, is denoted by ω and is shown by a double-headed vector using the right-hand screw convention. Each point of contact may now be viewed in terms of the co-ordinate system of a ball rolling on a plane shown in Fig. 1.

The rolling velocities are:

$$U_1 = \Omega_1 R, \quad U_2 = \Omega_2 R \quad . \quad . \quad (10)$$

To satisfy the condition of no sliding given by equation (2),

$$U_1 = r\omega\cos(\beta+\psi), \quad U_2 = r\omega\cos(\beta-\psi) \quad (11)$$

The angular velocity of spin at either point of contact is, by definition, the angular velocity component of the ball relative to the race about a normal axis through that point of contact, namely:

$$\left.\begin{aligned} \omega_{z1} &= \Omega_1\cos\beta+\omega\sin(\beta+\psi) \\ \omega_{z2} &= \Omega_2\cos\beta+\omega\sin(\beta-\psi) \end{aligned}\right\} \quad . \quad (12)$$

Making use of equations (10) and (11), the non-dimensional spin parameter $(\omega_z r/U)$ becomes in each case:

$$\left.\begin{aligned} \left(\frac{\omega_z r}{U}\right)_1 &= \frac{r}{R}\cos\beta+\tan(\beta+\psi) \\ \left(\frac{\omega_z r}{U}\right)_2 &= \frac{r}{R}\cos\beta+\tan(\beta-\psi) \end{aligned}\right\} \quad . \quad (13)$$

* *There is an apparent paradox in the fact that equation (9) has been obtained on the assumption of vanishingly small slip and yet implies a finite energy dissipated by slip. A comparable paradox is found in aerofoil theory. Viscosity (cf. slip) is postulated in order that the stagnation point should occur at the sharp trailing edge of the aerofoil and whereby circulation (cf. spin) is introduced. The lift and induced drag forces developed by the aerofoil are then calculated assuming the fluid to be inviscid. The external work done against the induced drag has then to be explained in terms of the kinetic energy dissipated in the trailing vortices.*

If conditions at the two points of contact are identical it might be expected that the ball would rotate about the axis of symmetry making the angle ψ zero. This supposition is subsequently examined.

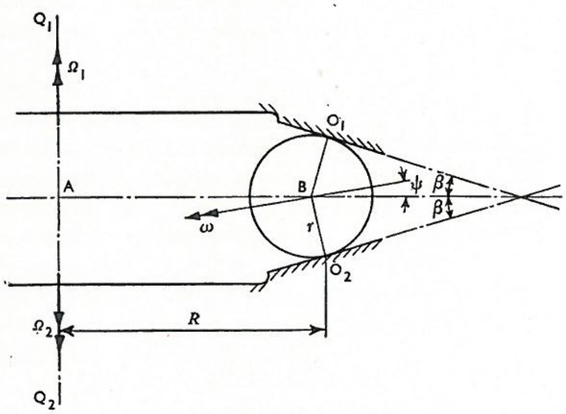
The ratio of angular speeds of the two discs follows from equations (10) and (11), and is given by:

$$\frac{\Omega_1}{\Omega_2} = \frac{\cos(\beta+\psi)}{\cos(\beta-\psi)} = \frac{1-\tan\beta\tan\psi}{1+\tan\beta\tan\psi} \qquad (14)$$

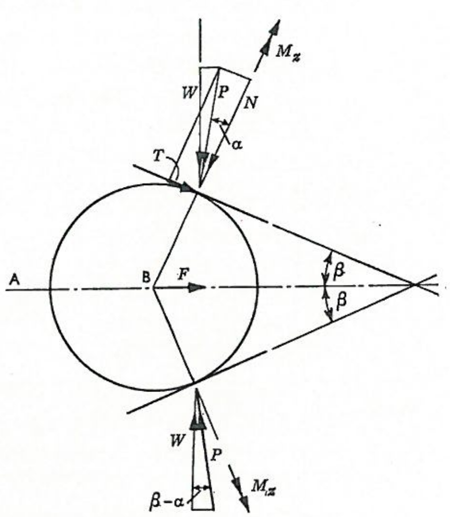
Equation (14) enables the angle ψ to be determined by observation of the velocity ratio Ω_1/Ω_2.

The angle β is, in fact, never large since it cannot exceed the angle of limiting friction. In most cases, therefore, it is justifiable to express equation (13) in approximate form:

$$\left.\begin{aligned} \left(\frac{\omega_z r}{U}\right)_1 &\simeq \frac{r}{R}+\beta+\psi \\ \left(\frac{\omega_z r}{U}\right)_2 &\simeq \frac{r}{R}+\beta-\psi \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$



a Kinematic aspect.



b System of forces.

Fig. 2. Rolling of a Ball Between Two Similar Concentric Discs

Turning now to the forces transmitted between the two discs through the ball, the ball carries a thrust load W parallel to the axis of rotation of the discs, and it is assumed that in addition it may be subject to a radial body force F*. The system of forces is shown in Fig. 2*b*. The resultant force at each point of contact is denoted by P, which is resolved into normal and tangential components N and T_y, where

$$\tan\alpha = T_y/N \quad . \quad . \quad . \quad . \quad (16)$$

For equilibrium of the ball,

$$P\cos(\beta-\alpha) = W$$

and

$$2P\sin(\beta-\alpha) = F$$

so that

$$\tan(\beta-\alpha) = \frac{F}{2W} \quad . \quad . \quad (17)$$

Expanding gives:

$$T_y/N = \tan\alpha = \frac{\tan\beta-\dfrac{F}{2W}}{1+\dfrac{F}{2W}\tan\beta} \quad . \quad (18)$$

The spin motion is opposed at each point of contact by a couple M_z acting about the normal axes. Resolving about the axis AB shows the necessity of a small tangential force T_x in the direction of rolling given by:

$$T_x = \frac{M_z}{r}\tan\beta$$

Although this force is too small to introduce any appreciable creep in the rolling direction, it contributes to the resistance to rotation of the discs. Resolving couples about the axis of rotation of the discs gives

$$M_\Omega = M_z\cos\beta + T_x R = M_z\left(\cos\beta+\frac{R}{r}\tan\beta\right) \qquad (19)$$

EXPERIMENTAL

Creep Measurements with Small Tangential Forces and Spin Velocities

In the previous section a theoretical expression has been given (equation (3)) for the transverse creep of a ball rolling on a plane surface under the combined action of a tangential force and an angular velocity of spin. This equation expresses the creep entirely in terms of the geometry of the ball and contact area, and the elastic properties of the surfaces. It has been derived on the assumption that the extent of slip within the contact area is vanishingly small and therefore can only be expected to apply when the magnitudes of the tangential contact force and the angular velocity of spin are small.

* *The experiments described here were performed at very slow rolling speeds so that the gyroscopic couple in the plane of Fig. 2b was negligible. The problem is not affected in principle by the presence of such a couple, but simplicity is forfeited in having a different force system at each point of contact. The radial force F was applied externally.*

In order to confirm the elastic analysis leading to equations (3) to (9) and to investigate the range in which they might be applied, an apparatus was constructed to measure the creep with considerable precision under the action of small tangential forces and spin, each of which could be varied independently. The apparatus is shown diagrammatically in Fig. 3.

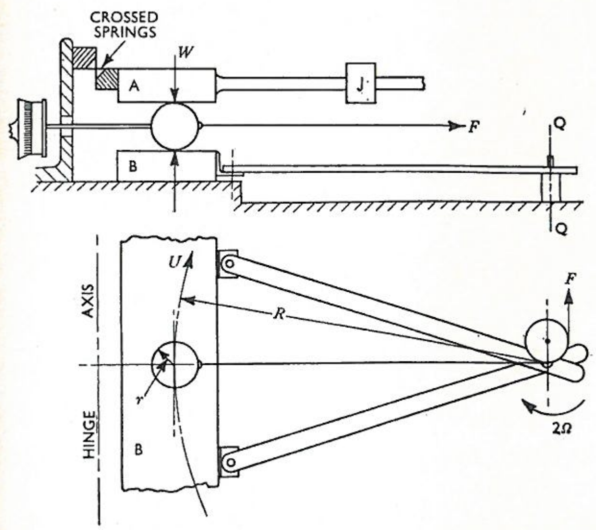


Fig. 3. Apparatus for Creep Measurements with Small Tangential Forces and Spin Velocities, Using a Single Ball Rolling Between Two Flat Surfaces

J	Jockey-weight.
A and B	Flats.
Q–Q	Fixed pivot.

A hard steel ball of optional size rolled between two hard steel flats A and B. The upper flat was hinged to a fixed support through a horizontal crossed-spring pivot, and the ball was loaded by the jockey-weight J. The lower flat rested on a smooth horizontal surface plate on which it was free to slide. It was constrained by two links to move through a short circular arc about the fixed vertical pivot Q–Q as centre. This sliding motion of the flat B was accompanied by rolling of the ball between the flats along an arc of radius R through half the distance moved by the flat. Rolling along a curved path of large radius introduced a small angular velocity of spin.

Observations showed that the ball turned about its radial axis of symmetry, so that the angle $\psi = 0$ (Fig. 2*a*). A force could therefore be applied to the ball through a thread attached to the ball on its axis of rotation. The lower flat was reciprocated by hand so that the ball rolled to and fro along an arc about 5 in. long. The direction of transverse creep resulting from both spin and the force F was independent of the rolling direction. Its effect therefore was cumulative, and a sufficient number of traversals of the arc could be performed until the radial displacement of the ball owing to its creep motion was large enough to be measured accurately by a sensitive micrometer (0·0001 in.).

As the ball rolled round an arc, its distance from the axis of the hinge changed slightly, thereby altering the load carried by the ball. For a short arc however this variation was negligibly small, particularly since the creep varies as the one-third power of the normal load.

Parallel Planes ($\beta = 0$) with External Force F

During the principal series of experiments with this apparatus the flats were set parallel to each other (that is, $\beta = 0$) and radially outward tangential contact forces were applied to the ball through the thread tension F.

Since ψ was observed to be zero, from equation (15) the spin parameters at both contact points were equal and given by:

$$\left(\frac{\omega_2 r}{U}\right)_1 = \left(\frac{\omega_2 r}{U}\right)_2 = \frac{r}{R}$$

The force F, being radially inward, was negative so that from equation (18),

$$\frac{T_y}{N} = \frac{F}{2W}$$

Taking the value of Poisson's ratio to be 0·3, equation (3) for the transverse creep becomes

$$\xi_y \frac{r}{a} = 0{\cdot}47\left(\frac{\omega_z r}{U}\right) - 0{\cdot}44\left(\frac{T_y}{N}\right) \quad . \quad (20)$$

A wide variation in the values of the spin parameter $(\omega_z r/U)$ was obtained by changing both ball size and track radius. At each value the creep was measured for progressively increasing values of F. A selection of the results of these tests is given in Fig. 4.

Equation (20) suggests that, other things being equal, the creep ratio ξ_y is proportional to the radius of the contact circle a (that is, proportional to $N^{1/3}$). The truth of this supposition has been confirmed in the author's earlier papers, but a further check was obtained in the current tests, with $r/R = 0{\cdot}012$, where creep measurements were made at loads of 15·5 lb. and 43·2 lb. (Fig. 4). Subsequent tests were carried out at one load only.

Referring to Fig. 4, it is clear that over most of the range of tangential force and spin covered by these experiments the transverse creep motion varies linearly with tangential force if the spin parameter is maintained constant. Conversely by cross plotting these results it appears that for constant values of (T_y/N) the creep varies linearly with spin. However, it may be seen that the lines in Fig. 4 are not parallel. Careful smoothing and analysis of these creep measurements shows that they can be represented closely by the empirical expression

$$\xi_y \frac{r}{a} = 0{\cdot}44\left(\frac{\omega_z r}{U}\right) - 3{\cdot}3\left(\frac{\omega_z r}{U}\right)\left(\frac{T_y}{N}\right) - 0{\cdot}55\left(\frac{T_y}{N}\right) \quad . \quad (21)$$

provided that $(\omega_z r/U) < 0{\cdot}06$ and $(T_y/N) < 0{\cdot}035$. The existence of the product term in this expression indicates that the effects of spin and force upon the creep motion are not completely independent as the theoretical equation (20) suggests. However, the product or 'coupling' term is relatively small in the operative range compared with the

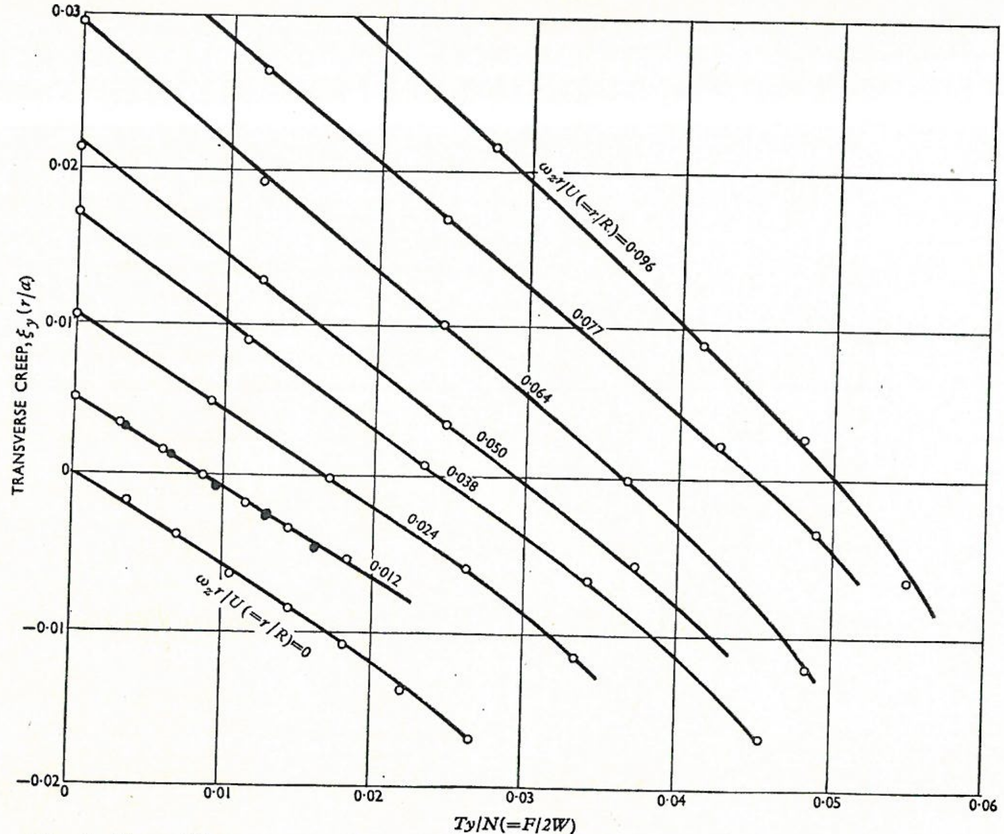


Fig. 4. Creep Measurements with Small Tangential Forces and Spin Velocities, from the Apparatus in Fig. 3

○ Load, $N = 43$ lb. ● $N = 15{\cdot}5$ lb.

other two terms whose coefficients 0·44 and −0·55 compare reasonably well with the theoretical values of 0·47 and −0·44 respectively.

An assessment of the discrepancy between the experimental results and the elastic theory may be obtained by considering the practically important case of the relationship between the tangential force and spin in order that the creep motion should be eliminated. This relationship is given theoretically by equation (8), which for $\nu = 0{\cdot}3$, becomes

$$\left(\frac{T_y}{N}\right)_{\xi=0} = 1{\cdot}07\left(\frac{\omega_z r}{U}\right)$$

The comparable empirical relationship is found by putting the left-hand side of equation (21) equal to zero. The comparison is shown in Fig. 5. The spot points are the actual intercepts with the horizontal axis of the lines of Fig. 4, thereby revealing the extent of smoothing involved in equation (21).

Straight Rolling Between Inclined Planes

When a ball rolls between two plane surfaces which are not parallel, spin is introduced even though the rolling path is a straight line. In addition, even with the body force $F = 0$, transverse tangential forces are introduced at the points of contact. This situation arises when a ball rolls in a straight V-groove.

For straight rolling ($R = \infty$) between planes inclined to each other at an angle 2β, and assuming rolling about the symmetrical axis ($\psi = 0$), the spin parameters given by equation (15) reduce to:

$$\left(\frac{\omega_z r}{U}\right)_1 = \left(\frac{\omega_z r}{U}\right)_2 = \tan\beta$$

In the absence of a body force F, equation (18) gives $T_y/N \equiv \tan\alpha = \tan\beta$. Thus, if the inclination of the planes is small, the resulting transverse creep, from equation (20), is given by:

$$\xi_y \frac{r}{a} \equiv 0{\cdot}47 \tan\beta - 0{\cdot}44 \tan\beta \simeq 0{\cdot}03\,\beta \qquad (22)$$

On the other hand the empirical expression of equation (21) reduces to

$$\xi_y \frac{r}{a} = -0{\cdot}11\,\beta\,(1+30\,\beta) \qquad . \quad (23)$$

The difference between equations (22) and (23) raises a point of practical importance. If the creep is positive, as in equation (22), and the two planes forming the groove are a fixed distance apart, then as rolling proceeds the ball will wind itself tighter and tighter into the groove until

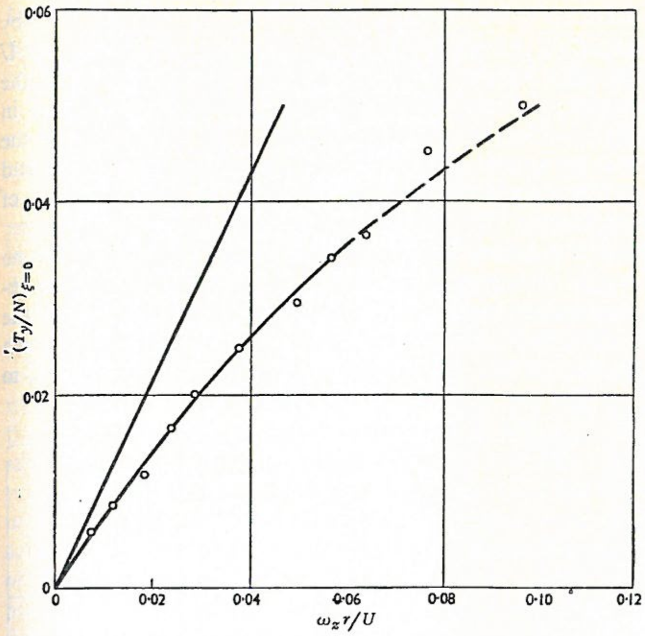


Fig. 5. Relationship Between Tangential Force and Spin for Zero Creep Motion

——— Theoretical equation (8).
○ Empirical equation (21) $\xi_y = 0$.

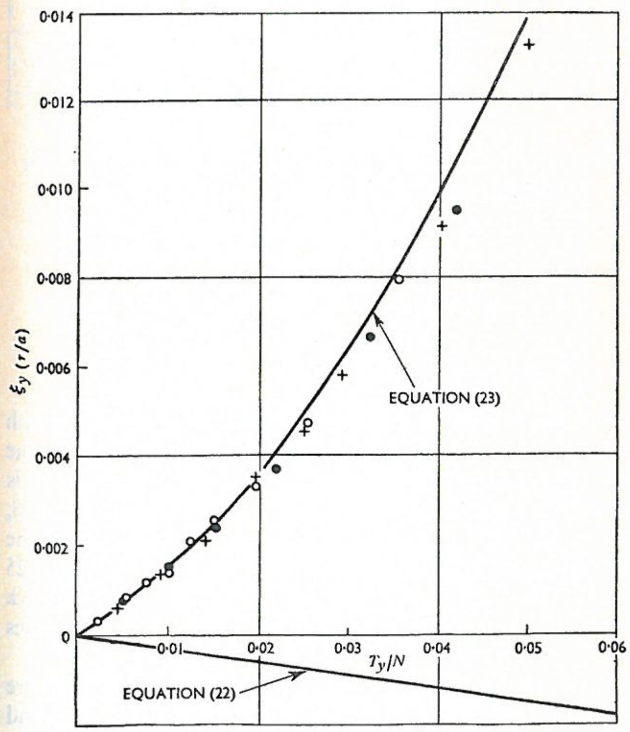


Fig. 6. Creep During Straight Rolling Between Inclined Planes

$F = 0,\ T_y/N = \beta,\ \omega_z r/U = \beta.$

● $r = 0{\cdot}094$ in., $N = 16$ lb.
+ $r = 0{\cdot}375$ in., $N = 19$ lb.
○ $r = 0{\cdot}375$ in., $N = 35$ lb.

frictional resistance prevents further rolling. On the other hand negative creep, as given by equation (23), would ensure that the ball does not become jammed in this way.

The apparatus shown in Fig. 3 was arranged to measure the creep in straight rolling with the two flats slightly inclined to each other. The lower flat B was detached from the pivot Q and was guided to slide backward and forward in a straight line. By adjusting the height of the hinge of the upper flat A the included angle between them 2β could be given any required small value. The creep was measured as before and was found to be completely consistent with the previous experiments, as defined by equation (23). The observations are compared with equations (22) and (23) in Fig. 6. Due to the fact that in this case the two terms of equation (20) are of almost equal magnitude, relatively small discrepancies between experiment and theory make the theoretical equation (22) completely unreliable. The experimental results, however, are all self-consistent, and it appears therefore that the empirical expression of equation (21) may be used to calculate the creep velocities however the actions of spin and tangential force arise.

Creep Measurements with Large Tangential Forces and Spin Velocities

Under the action of small tangential forces and spin velocities the creep motion is determined almost entirely by the elastic properties of the rolling solids. As the forces and spin are increased extensive slip begins to take place within the contact area so that the creep is now influenced by the surface conditions.

To extend the results of the previous section (shown in Fig. 4) beyond the range in which equation (21) applies, creep measurements were made using a different experimental arrangement. The apparatus took the form of a simple thrust bearing consisting of two identical hard-steel circular discs with their axis of rotation vertical. The lower disc was fixed whilst the upper disc, free to rotate about its axis, transmitted a dead load through three equally spaced balls. The circular tracks of the balls were shallow grooves, of about 3 in. radius, ground in the faces of the discs. As the upper disc turned the balls rolled along an ostensibly circular path of radius R. The relatively large radius of curvature of the groove, that is, small conformity of balls and tracks, ensured that the area of contact was very nearly circular so that the conditions at the points of contact were comparable with those of a ball rolling between plane surfaces.

In the analysis of rolling bearings it is usual to assume that the balls take up a position in which the tangent planes at the two points of contact are parallel, which would correspond in this case to the balls rolling in the trough of the grooves where the tangent planes are horizontal. But this is not the only position of equilibrium. Obviously the balls could roll without sliding along any track in which the tangent planes at the points of contact were inclined to each other by an angle not greater than twice the limiting angle of friction for the surfaces. Suppose that the balls started in the trough of the grooves, with the tangent planes parallel, the previous investigation would lead to the expectation that as rolling proceeded the balls would creep radially outward due to the action of spin and thereby would climb the sides of the grooves until tangential forces were brought into play at the points of contact as shown in Fig. 2. Their effect would be to reduce the radial creep until a 'steady state' was reached in which the balls roll along an unvarying circular path with the tangent-planes inclined at an angle $2\beta_s$. Rolling would then proceed without any further creep, which is the situation expressed by equations (8) and (9) and in Fig. 5.

The procedure followed in these experiments was to start with the balls at a radius slightly less than that of the trough of the groove (that is, with a negative angle β) and to measure the radial creep of the balls for each revolution round their track until the steady-state path was approached. The balls were then started at a radius greater than that of the steady-state path, and their radially inward creep was measured. The steady-state inclination β was thus approached from below and above. From the geometry of the ball and tracks, the radial position of the ball at any instant gave an accurate measure of the inclination β.

It was again assumed *a priori* that the conditions appertaining at each point of contact were the same, so that the angle ψ could be taken to be zero. The spin parameters at each point of contact were therefore given by equation (13) or approximately by equation (15) ($\psi = 0$). At slow rolling velocities there were no appreciable body forces ($F = 0$) so that, in this case, the force parameter given by equation (18) reduced to $T_y/N = \tan\beta$.

The measurements described above were carried out for a range of ball diameters ($\frac{1}{4}$ in. to 2 in.) and for two different pairs of discs ($R = 1{\cdot}48$ and $3{\cdot}47$ in.) thereby obtaining a wide variation in the spin parameter $\omega_z r/U$ ($= r/R \cos\beta + \tan\beta$). After analysis and smoothing, the results of these experiments are shown in the chart in Fig. 7. The creep parameter $\xi(r/a)$ is a function of the spin parameter $\omega_z r/U$ and the force parameter T_y/N, and is shown plotted against T_y/N for constant values of $\omega_z r/U$. Fig. 7 is similar to and extends Fig. 4.

The relation between tangential force and spin for no creep, corresponding in these experiments to the steady-state rolling path, is obtained by the intercepts on the T_y/N axis of the lines in Fig. 7. This relationship is shown by the full line in Fig. 8 which provides an extension, to

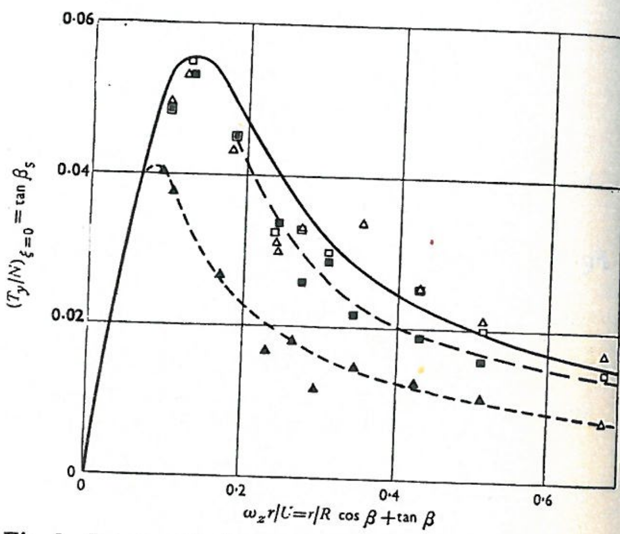


Fig. 8. Relationship Between Tangential Force and Spin for Zero Creep Motion (Fig. 5 extended)

——— Smoothed results of Fig. 7 (dry).
□ Surfaces dry, rolling slowly.
△ Surfaces dry, rolling fast.
■ Surfaces lubricated, rolling slowly.
▲ Surfaces lubricated, rolling fast.

a very different scale, of Fig. 5. Physically this graph represents the inclination of the tangent planes when the steady-state condition is reached. As the effect of spin is increased it may be seen that the angle of inclination β_s reaches a maximum value of about 0·055 (about half of the angle of limiting friction) at a value of $\omega_z r/U \simeq 0{\cdot}125$ which would occur when the ratio of ball radius to track radius, $r/R \simeq 0{\cdot}07$. For balls of larger size the spin increases and β_s decreases.

In the range of large spin velocities ($\omega_z r/U > 0{\cdot}1$) there is considerable slip taking place in the contact area and slight differences in the condition of the surfaces: roughness, the presence of a lubricant, and so on, affect the creep. There is much more scatter, therefore, in the results of these experiments than in those described in the previous section, as a comparison of the observations in Figs. 5 and 8 shows.

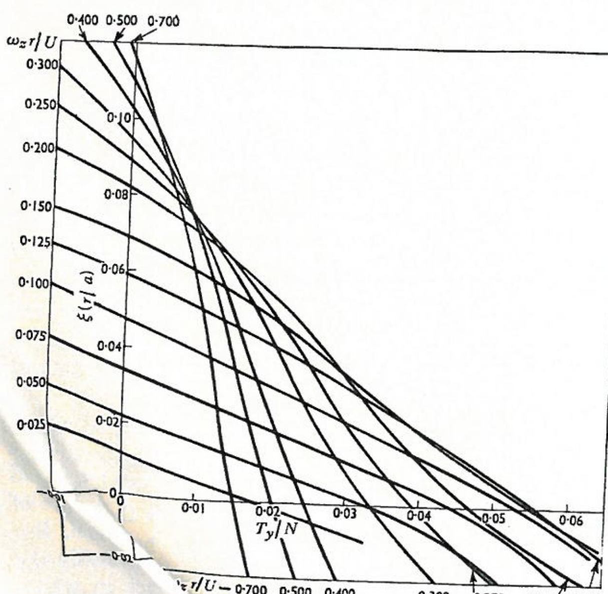


Fig. 7. Creep ... Forces and Spin ...surements with Larger Tangential ...$_y$(r/a) the Thrust Bearing Apparatus ...$\omega_z r/U$, T_y/N).

Frictional Resistance to Rolling with Spin

Where rolling elements are used in engineering devices it is usually with the purpose of reducing frictional resistance to motion. A fundamental study of the rolling process is not complete, therefore, without an examination of the nature and magnitude of the frictional resistance.

It has been recognized for some time that two distinct processes contribute to 'rolling friction' (*a*) relative slip between the surfaces and (*b*) dissipation of energy within the rolling solids by the so called 'elastic hysteresis' of their material. It has been claimed (2) that, even in 'free rolling' (that is, a purely normal force and no spin), when a ball rolls on a plane or, more particularly, in a groove 'pure' rolling can only occur along two contours of the ball, and that elsewhere in the contact area relative slip must be taking place. This theory, whilst admitting Hertzian compression of the ball and track, does not admit their capacity to distort elastically in the *tangential* direction under the action of friction forces and thereby prevent slip. The work of Palmgren (3) and more recently that of Tabor (4), however, has shown that very little slip does take place in free rolling and that the major contribution to rolling friction arises from the elastic hysteresis of the material.

Tabor's experiments have been further confirmed by calculations made by the author (5) of the frictional tractions necessary to eliminate slip in free rolling.

The introduction of either tangential contact forces or a spin motion, on the other hand, does cause slip between the rolling surfaces over at least part of their contact area. The slip starts at the trailing edge of the contact circle and spreads forward as the magnitude of the tangential force or spin velocity is increased. This pattern of slip is illustrated clearly in Fig. 9. The photographs are of the contact area of a rubber ball rolling on a perspex plate on to which there had been printed previously a screen of ink dots. The ink was still wet and therefore smeared in the direction of relative slip.

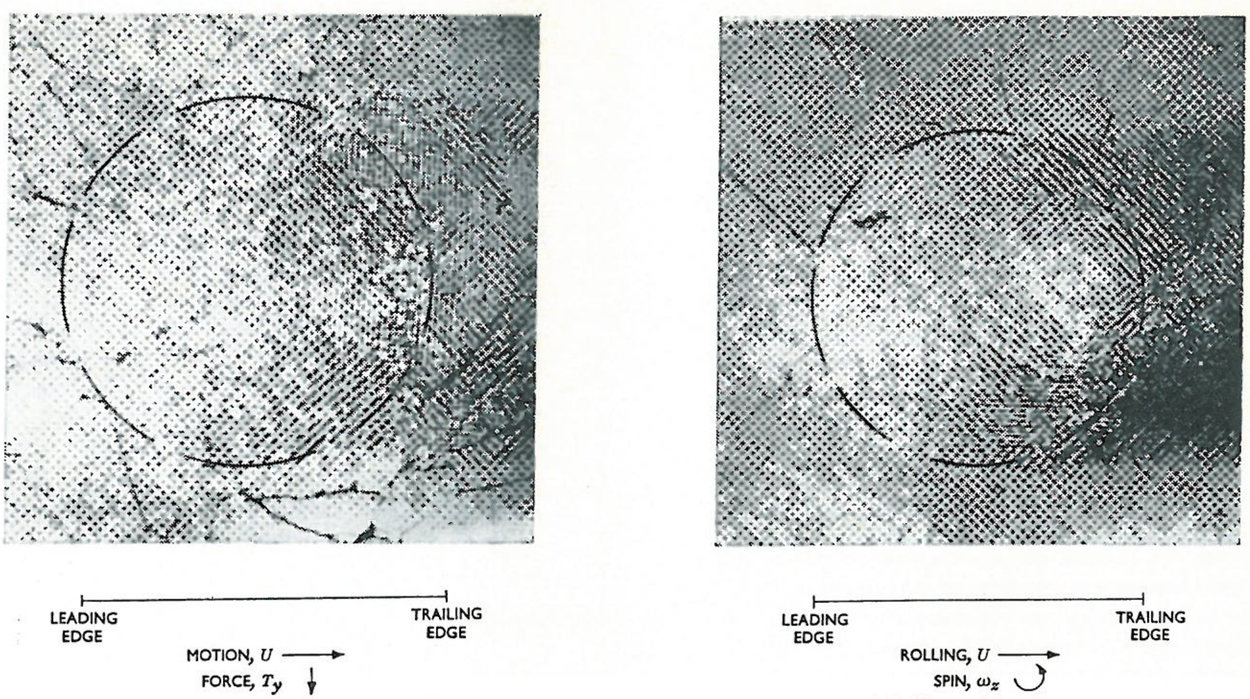
In these circumstances the overall resistance to rolling is made up of the contribution due to the elastic hysteresis of the material together with the effect of slip arising from the action of tangential forces and spin. On the basis of Tabor's elastic hysteresis theory (4) it has been found possible to separate the two effects (1b). The three-ball apparatus used in the experiments just described enables the overall rolling resistance to be measured by observing the friction couple resisting the rotation of the upper disc about its axis Q–Q. The total moment about this axis is made up of two terms, one due to hysteresis and the other due to spin; thus:

$$\left(\frac{M_\Omega}{Na}\right)_{\text{total}} = \left(\frac{M_\Omega}{Na}\right)_{\text{hys.}} + \left(\frac{M_\Omega}{Na}\right)_{\text{spin}} = k\frac{R}{r} + f\left(\frac{\omega_z r}{U}, \frac{T_y}{N}\right) \quad . \quad . \quad . \quad . \quad (24)$$

where k is a constant depending upon the elastic hysteresis of the material. If the balls are rolling between parallel races ($\beta = 0$), equations (19) and (13) give $(M_\Omega/Na)_{\text{spin}} = M_z/Na = f(r/R)$ whence

$$\frac{r}{R}\left(\frac{M_\Omega}{Na}\right)_{\text{total}} = k + f_1\left(\frac{r}{R}\right) \quad . \quad . \quad (25)$$

The total moment resisting rotation of the upper disc M_Ω was measured by observing the retardation of the disc when rotating freely, with the friction of the centre spindle



a Rolling with transverse tangential force.

b Rolling with spin.

Fig. 9. Patterns of Slip

reduced to a minimum. Measurements at three different normal loads enabled mean values for (M_Q/Na) to be obtained for different ratios of ball to track radius r/R. Plotting in the form of equation (25) and extrapolating to $r/R = 0$ gave a value for $k = 0{\cdot}0018$, which is consistent with Tabor's hysteresis measurements. The moment due to spin could then be found by subtraction. These results are given in the upper curve in Fig. 10.

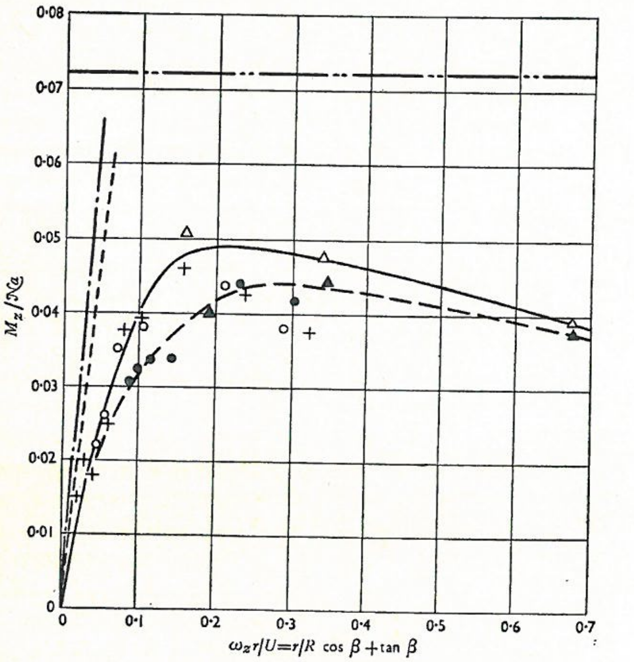


Fig. 10. Frictional Resistance to Rolling with Spin

Found by subtracting the contribution due to elastic hysteresis from the overall resistance.

- ——— Rolling between parallel planes, $\beta = 0$.
- — — — Steady-state rolling (no creep, $\xi = 0$, $\beta = \beta_s$).
- - - - - - Equation (9).
- —-— Equation (7).
- —-—- Resistance to spin without rolling.
- ○ Rolling between parallel planes, $R = 3{\cdot}5$ in.
- △ Rolling between parallel planes, $R = 1{\cdot}5$ in.
- ● Steady-state rolling, $R = 3{\cdot}5$ in.
- ▲ Steady-state rolling, $R = 1{\cdot}5$ in.

Having found a value for k which defines the hysteresis component, the total friction moment in the steady-state rolling condition $(\beta = \beta_s)$ could be measured and the spin moment M_z deduced by using equation (19). These results are given in the lower curve in Fig. 10.

The spin moment M_z has been calculated theoretically for both cases, that is, when $\beta = 0$ and when $\beta = \beta_s$, and is given by equations (7) and (9) which are also shown in Fig. 10. The measured moments, although rather approximate, are appreciably less than the theoretical. Presumably the effect of even a small amount of slip, in reducing the peak values of the theoretical surface tractions given by equations (5) and (6), has a marked effect upon the resisting moment due to those tractions.

Previous theoretical estimates of the friction moment resisting rolling with spin (6) and (7) have been based on the assumption that the relative motion between the surfaces in contact was one of a simple spin rotation (without rolling), about the centre of the contact area. Slip in a circumferential direction was assumed to occur at all points in the contact region except at its centre. The spin moment is entirely frictional and is given by:

$$M_z = \frac{3\pi}{16}\mu N a \quad . \quad . \quad . \quad . \quad (26)$$

An experiment was performed to simulate the situation expressed by equation (26). The friction moment opposing spin of a ball without rolling was measured under the same surface conditions as in the rolling experiments and was found to give the consistent value, $M_z/Na = 0{\cdot}072$. By equation (26) this gives $\mu = 0{\cdot}12$, which agrees with previous observations. It is immediately apparent from Fig. 10 that even the maximum resistance during rolling is considerably less (68 per cent) than the resistance to simple spin. This is only to be expected when it is recollected that, during rolling, slip occurs over only part of the contact area, and that the direction of slip is by no means everywhere in the circumferential direction.

A further feature of Fig. 10 calls for comment. As the spin parameter increases, the moment M_z increases to a maximum and then shows a tendency to decrease. A decreasing moment leads to uncertainty about the axis of rotation of the ball. Referring to Fig. 2*b*, equilibrium of the ball demands that the spin moments at each point should be equal. Over the portion of Fig. 10 in which the moment increases monotonically this condition is satisfied by the spin parameters being equal at each point of contact, from which, by equation (13), it follows that $\psi = 0$. The ball, therefore, turns about its radial axis of symmetry. To the right of Fig. 10, where it appears that the moment decreases with increasing spin, this state of affairs no longer holds. Any slight difference between the conditions at the two points of contact causes the axis of rotation of the ball to depart from the axis of symmetry, thereby increasing the effective spin parameter at O_1 and correspondingly decreasing it at O_2, as specified by equation (13). The value of ψ will be such that the spin moments at O_1 and O_2 are still equal, with M_{z2} on the rising part of Fig. 10 and M_{z1} on the falling part. It should be noted that Fig. 10 itself has been plotted assuming that $\psi = 0$, that is, assuming $(\omega_z r/U)_1 = (\omega_z r/U)_2 = r/R \cos\beta + \tan\beta$. By the above argument the points on the falling part of the curve should bifurcate, one moving left on to the rising curve and the other moving an approximately equal distance to the right. By such a construction the expected values of ψ may be estimated. In any case, it would appear that the observations on the falling part of Fig. 10 are uncertain and should in all probability lie further to the right.

Observations of the Rolling Axis of the Ball

Clearly some investigations had to be made of how the rolling axis of the ball varied, if at all, as the spin parameter was increased.

It has already been mentioned that for small spin velocities observation of a spot marked on the axis of symmetry of the ball showed that the ball turned about that axis (that is, $\psi = 0$) with negligible variation. For values of $\omega_z r/U$ greater than about 0·15 this was noticed to be no longer the case. Direct observation of the rolling axis proved unsatisfactory since it appeared that the value of ψ varied from time to time.

With the ball rolling in its steady-state path, however, so that $\beta \neq 0$, the effect of an inclination of the rolling axis is to make the angular velocity of the two discs about their common axis unequal (equation (14)). Precise measurement of this difference presented no difficulties and hence observations of the mean value of ψ associated with steady-state rolling were made for the range of spin parameters covered by the previous experiments. It was found that ψ was particularly sensitive to surface conditions, so that care was taken to polish the rolling tracks of each disc to approximately the same surface finish, and observations were made with the surfaces dry and lubricated, also rolling slowly and (by the standard of these experiments) fast.

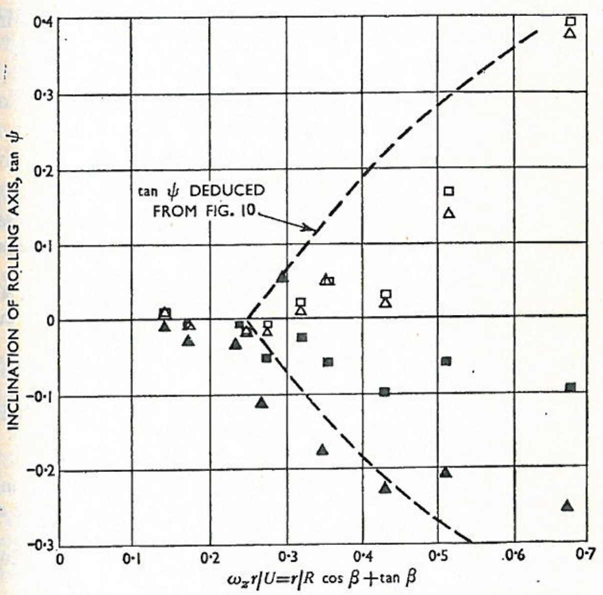


Fig. 11. Inclination of the Axis of Rotation of a Ball Rolling Between Two Discs

□ Surfaces dry, rolling slowly.
△ Surfaces dry, rolling fast.
■ Surfaces lubricated, rolling slowly.
▲ Surfaces lubricated, rolling fast.

The results are shown in Fig. 11. The scatter in the value of ψ is immediately apparent. This is the result of the very gradual decrease of spin moment with spin on the right-hand side of Fig. 10 together with the sensitivity of ψ to surface conditions. These two facts combine to make ψ effectively indeterminate within certain bounds. These bounds correspond roughly to those found from an analysis of Fig. 10. We should expect ψ to be zero provided that $\omega_z r/U$ is less than the value corresponding to the maximum moment in Fig. 10, in this case 0·25 for steady-state rolling with dry surfaces. And, in fact, ψ was observed to be less than $\pm 1°$ in this range. For greater values of $\omega_z r/U$, ψ will not be zero, but its maximum value is limited to that which would reduce the value of $(\omega_z r/U)_2$, given by equation (13), to a point on the rising part of Fig. 10, that is, to the left of the maximum. Bounds to the value of ψ estimated in this way are shown in Fig. 11. They are only exceeded by observations made with fast rolling on lubricated surfaces, under which conditions (not measured) the peak of Fig. 10 is likely to be moved to the left.

CONCLUSIONS

When a ball rolls between two surfaces, tangential forces are transmitted at the points of contact and, further, there is a relative angular velocity of spin between the surfaces in contact. Both the tangential contact force and the spin motion induce tangential tractions at the contact interface which are maintained by friction. At the trailing edge of the contact area, where the tangential tractions are high, slipping begins and spreads forward across the contact area as the applied tangential force or the amount of spin is increased.

The elastic deformation and slip in the contact region affect the motion of the rolling ball by the introduction of a small creep velocity at right angles to the nominal rolling path. It has been shown that this effect may be expressed in the non-dimensional form:

$$\frac{\text{transverse creep velocity}}{\text{forward rolling velocity}} \equiv \xi_y = \frac{a}{r} f\left\{\left(\frac{\omega_z r}{U}\right), \left(\frac{T_y}{N}\right), \mu\right\} \quad . \quad . \quad . \quad (27)$$

where the non-dimensional parameters $\omega_z r/U$ and T_y/N control the influence of spin and tangential force respectively, and μ is some measure of the frictional properties of the contact surfaces. The above expression specifies the creep velocity at one point of contact only. To prescribe the motion of the ball completely it is necessary to determine the distribution of spin velocity between the two points of contact, that is, to determine the axis about which the ball rolls.

The investigation has shown that the behaviour of the ball predominantly depends upon whether the spin velocities are small or large.

If the spin and force parameters are both small ($\omega_z r/U < 0{\cdot}06$ and $T_y/N < 0{\cdot}035$) the surfaces roll together without slipping over most of the area in which they are in contact. Slip is confined to a thin region at the trailing edge. In these circumstances the motion of the ball is precisely defined; the creep depends upon the elastic properties of the materials and not to any appreciable extent upon frictional properties of the surfaces, provided that the surface asperities are small compared with the dimensions of the contact area. The axis of rotation of the ball takes up a position which makes the spin parameters equal at each point of contact. The creep motion is then given by the results in Fig. 4 or the empirical equation (21).

The theoretical expressions for the creep ratio (equation

(3)), based on the assumption that there is no slip at all, agrees roughly with the experiments. It is valuable in showing how the creep motion might be expected to vary with elastic constants G and ν. The agreement also suggests that the theoretical distributions of tangential surface tractions (equations (5) and (6)) are reasonably correct except close to the boundary of the contact circle.

When the spin motion is relatively large, slip extends over an appreciable proportion of the contact area, and in consequence the creep motion is influenced by the frictional properties of the surfaces. Making the simple hypothesis that slip at any point is governed by Amonton's Law, with a constant coefficient of friction μ, then it can be shown (1) that equation (27) may be written:

$$\xi_y = \frac{\mu a}{r} f\left\{\left(\frac{\omega_z r}{U\mu}\right), \left(\frac{T_y}{\mu N}\right)\right\} \quad . \quad . \quad (28)$$

The experiments have shown that this statement is an oversimplification. It has been observed that the creep velocity is influenced by surface finish, lubrication, and speed of rolling, but not in any simple way which can be associated with the coefficient of friction as measured in a 'sliding' experiment. These factors have not been investigated systematically, but a few observations can usefully be made.

The creep (at large spin velocities) is sensitive to surface finish and may be reduced by 30–40 per cent by polishing the rolling surfaces. The results in Figs. 6–11 were obtained using surfaces whose average roughness was 3·5 μin. c.l.a. (centre line average). The importance of surface roughness is less surprising when it is remembered that the relative displacements comprising 'slip' are less than 10^{-4} in., which is of the same order as the asperity heights.

The influence of a lubricant (light machine oil) is closely related to that of rolling speed. With the surfaces clean and dry the creep is unaffected by the speed of rolling within the range of low speeds used in these experiments (< 20 ft/min). Slip appears to be facilitated by the presence of a lubricant thereby reducing the creep (Fig. 8). But, provided that the speed is sufficiently low (about 2 ft/min), the lubricant is squeezed out and the creep measurements differ little from those obtained with dry surfaces.

At higher spin velocities the axis of rotation of the ball becomes indeterminate and therefore it is no longer correct to assume that the spin motion is shared equally by the two contact areas. Differences in surface finish or conformity between the ball and tracks would cause the majority of the spin, and hence slip, to take place at one of the points of contact.

As generally recognized the spin motion produces a resistance to rolling. The magnitude of this resistance varies with the spin parameter $\omega_z r/U$ and is appreciably lower than previous estimates based upon complete slip.

In conclusion the limitations of the investigation must be remembered. Firstly, at high rolling speeds such as are common in ball-bearings, for example, hydrodynamic action between the rolling surfaces is likely to be a significant factor. Secondly, a high degree of conformity between the ball and the surfaces on which it rolls, so that the contact area is markedly elliptical, would probably make appreciable quantitative changes to the results quoted. But neither of these factors is likely to alter the qualitative picture of the rolling process which has been revealed by this investigation.

ACKNOWLEDGEMENT

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APPENDIX

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