Example 5d: Fatigue Damage Analysis

MAC/GMC 4.0 includes two fatigue damage models that function on the local subcell level, both of which are employed in this example problem. The first involves degradation of the subcell material stiffness properties due to applied cyclic loading (suitable for modeling the composite matrix behavior), while the second involves degradation of the subcell material strength properties (suitable for modeling the composite fiber behavior). The same 60% fiber volume fraction graphite/epoxy composite is again considered as both a unidirectional composite and a quasi-isotropic laminate.

The first step in this example problem involves characterizing the epoxy and graphite phases in terms of the fatigue damage model parameters. First, considering the epoxy matrix, the stiffness degradation fatigue damage model will be employed. The transversely isotropic form of this damage model present within MAC/GMC 4.0 is multi-axial, isothermal, and it employs a single scalar internal damage parameter, D (Arnold and Kruch, 1994). This damage parameter begins at zero for an undamaged material and grows (for a particular subcell) as damage occurs due to cyclic loading. The stiffness of the material in a damaged subcell is reduced by a factor of $(1 - D)$ to account for the damage. A value of D = 1 corresponds to the completely damaged (zero stiffness) case. When employed (as in this example) to model the fatigue behavior of the isotropic material present within a particular subcell, this model reduces to the NonLinear Cumulative Damage Rule (NLCDR) developed at ONERA (Wilt et al., 1997). For details on this fatigue damage model, see the MAC/GMC 4.0 Theory Manual Section 5.3.

For an isotropic material, the damage parameters that must be selected reduce to M, β and $\hat{a}$, and the pertinent equation relating the fatigue life of the isotropic material to the cyclic stress state is,

$$N_F = \frac{(\sigma_u - \sigma_{\max})\left(\dfrac{M}{\sigma_{\max} - \bar{\sigma}}\right)^{\beta}}{\hat{a}(1+\beta)(\sigma_{\max} - \bar{\sigma} - \sigma_{fl})} \qquad for\ N_F > 0$$

where σ_u is the material ultimate strength, σ_{fl} is the material fatigue limit (stress below which damage does not occur), $\sigma_{\max}$ is the maximum stress during a loading cycle, $\bar{\sigma}$ is the mean stress during a loading cycle, and N_F is the number of cycles to failure. Note that, in the terminology of Arnold and Kruch (1994), $\hat{a} = a\dfrac{\sigma_u}{\sigma_{fl}}$. Utilizing the above equation, the damage model parameters M, β and $\hat{a}$ can be selected for an isotropic material based on the material's S-N curve (stress level vs. cycles to failure). A suggested characterization procedure for this damage model can be found in Arnold and Kruch (1994).

An S-N curve for epoxy was obtained from Plastics Design Library (1995), and the fatigue damage model parameters were selected as $M = 150$ MPa, $\beta = 9$, and $\hat{a} = 0.05$, with $\sigma_u = 80$ MPa, and $\sigma_{fl} = 27$ MPa. A plot showing the fatigue model characterization is given in Figure 5.5.

The second damage model within MAC/GMC 4.0 is much simpler and involves degradation of a material's strength due to cyclic loading. As shown by Wilt et al. (1997), this type of damage model can be used to simulate the fatigue behavior of fibers that occurs in-situ during fatigue of a composite.

The model assumes a logarithmic relation between the material's strength and the number of cycles within a certain range such that:

$$\sigma_u = \sigma_{u1} \qquad 0 \le N \le N_1$$

$$\sigma_u = \sigma_{u1} - \frac{(\sigma_{u1} - \sigma_{u2})\log(N/N_1)}{\log(N_2/N_1)} \qquad N_1 \le N \le N_2$$

$$\sigma_u = \sigma_{u2} \qquad N_2 \le N$$

This strength degradation model was employed in the present example to model the longitudinal fatigue behavior of the graphite fiber. The necessary parameters for the model are σ_{u1}, σ_{u2}, N_1, and N_2. The values of these parameters chosen for the graphite fiber are shown in Figure 5.6. Note that these data were not correlated with experiment, but rather chosen based on the expected trend.

Given these required parameters for the fatigue damage models for each phase in the graphite/epoxy composite, this example problem, which predicts the fatigue life of a composite and a laminate, can be executed.

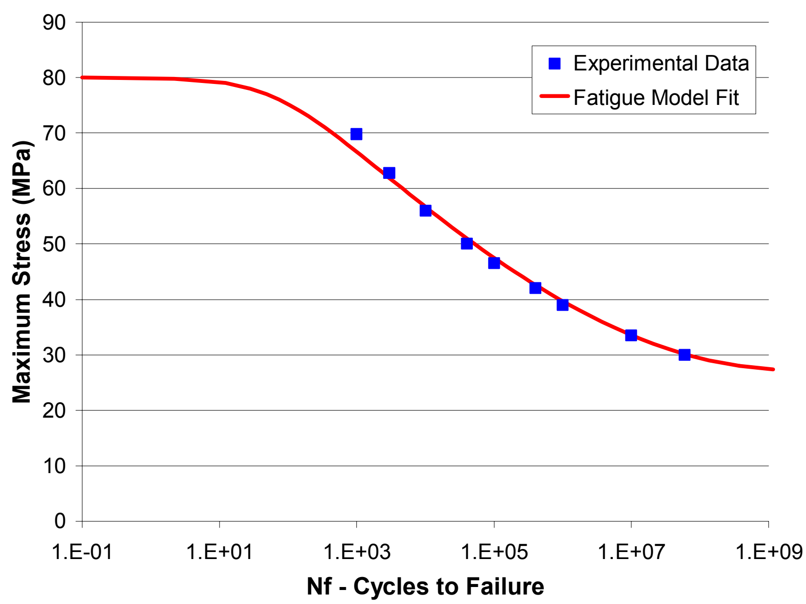

Figure 5.5 Example 5d: Characterization of the stiffness reduction fatigue damage model parameters for the epoxy matrix. Experimental data are from Plastics Design Library (1995).

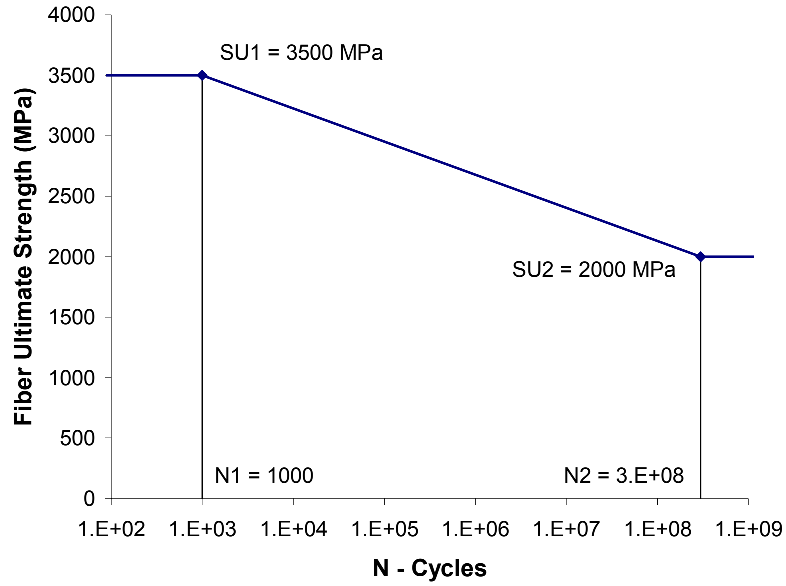

Figure 5.6 Example 5d: Strength reduction fatigue model parameters assumed for the graphite fiber.

MAC/GMC Input File: `example_5d.mac`

```
MAC/GMC 4.0 Example 5d - Fatigue damage analysis
*CONSTITUENTS
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=388.2E3,390.E3
  ET=7.6E3,7.6E3
  NUA=0.41,0.41
  NUT=0.45,0.45
  GA=14.9E3,15.1E3
  ALPA=-0.68E-6,-0.45E-6
  ALPT=9.74E-6,10.34E-6
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=3.45E3,3.10E3
  ET=3.45E3,3.10E3
  NUA=0.35,0.35
  NUT=0.35,0.35
  GA=1.278E3,1.148E3
  ALPA=45.E-6,55.E-6
  ALPT=45.E-6,55.E-6
*RUC
  MOD=2 ARCHID=6 VF=0.60 R=1. F=1 M=2
#*LAMINATE
# NLY=7
#  LY=1 THK=0.125 ANG=0   MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
```

```
#  LY=2 THK=0.125 ANG=45  MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
#  LY=3 THK=0.125 ANG=-45 MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
#  LY=4 THK=0.250 ANG=90  MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
#  LY=5 THK=0.125 ANG=-45 MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
#  LY=6 THK=0.125 ANG=45  MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
#  LY=7 THK=0.125 ANG=0   MOD=2 ARCHID=6 R=1. VF=0.60  F=1 M=2
*MECH
  LOP=2
#  LOP=1
  NPT=4 TI=0.,50.,150.,200. MAG=0.,40.,-40.,0. MODE=2,2,2
*THERM
  NPT=4 TI=0.,50.,150.,200. TEMP=23.,23.,23.,23.
*SOLVER
  METHOD=1 NPT=4 TI=0.,50.,150.,200. STP=10.,10.,10.
*DAMAGE
 MAXNB=100 DINC=0.2 DMAX=1.0 BLOCK=0.,200.
 NDMAT=2
 MAT=1 MOD=2 SU1=3500,91.2,91.2,31.4,134.,134 &
             SU2=2000.,91.2,91.2,31.4,134.,134. &
             N1=1000,1000,1000,1000,1000,1000 &
             N2=300000000,300000000,300000000,300000000,300000000,300000000
 MAT=2 MOD=1 ANG=0. BN=0.0 BP=0.0 OMU=1. OMFL=1. OMM=1. ETU=1. &
       ETFL=1. ETM=1. BE=9. A=0.05 SFL=27. XML=150. &
       SU=80.
*FAILURE_SUBCELL
  NMAT=2
  MAT=1 NCRIT=1
   CRIT=1 X11=3500. X22=91.2  X33=91.2  X23=31.4  X13=134.  X12=134.  &
    COMPR=SAM
  MAT=2 NCRIT=1
   CRIT=1 X11=80.   X22=80.   X33=80.   X23=40.   X13=40.   X12=40.   &
    COMPR=SAM
*FAILURE_CELL
  NCRIT=1
  CRIT=2 X11=0.05 X22=0.05  X33=0.05  X23=0.05  X13=0.05 X12=0.05  &
    COMPR=SAM
*PRINT
  NPL=3
*XYPLOT
  FREQ=1
#  LAMINATE=1
#   NAME=example_5d X=1 Y=10
  MACRO=2
   NAME=example_5d_11 LYR=1 X=1 Y=7
   NAME=example_5d_22 LYR=1 X=2 Y=8
  MICRO=0
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

Number of materials:	2	(NMATS=2)
Constitutive models:	Elastic	(CMOD=6)
Materials:	User-defined (Graphite)	(MATID=U)
	User-defined (Epoxy)	(MATID=U)
Material property source:	Read from input file	(MATDB=1)
Material properties:	See Table 4.1	

3) Analysis type:

a) Unidirectional Composite Case (***RUC**) → Repeating Unit Cell Analysis [KM_3]:

Analysis model:	Doubly periodic GMC	(MOD=2)
RUC architecture:	7×7 circular fiber approx., rect. pack	(ARCHID=6)
Fiber volume fraction:	0.60	(VF=0.60)
Unit cell aspect ratio:	1.0 (square pack)	(R=1.0)
Material assignment:	graphite fiber	(F=1)
	epoxy matrix	(M=2)

b) Quasi-Isotropic Laminate Case (***LAMINATE**) → Laminate Analysis [KM_3]:

Number of layers:	7	(NLY=7)

Layer	Analysis Model	Thickness	Fiber Angle	Architecture	Volume fraction	Aspect ratio	Fiber material	Matrix material
(LY=)	(MOD)	(THK)	(ANG)	(ARCHID)	(VF)	(R)	(F)	(M)
1	GMC-2D	0.125	0°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
2	GMC-2D	0.125	45°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
3	GMC-2D	0.125	-45°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
4	GMC-2D	0.25	90°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
5	GMC-2D	0.125	-45°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
6	GMC-2D	0.125	45°	7×7 circle rect. pack	0.60	1.	graphite	epoxy
7	GMC-2D	0.125	0°	7×7 circle rect. pack	0.60	1.	graphite	epoxy

☞ Note: To generate the data in the Results for both the unidirectional composite and the quasi-isotropic laminate, the appropriate lines in the input file must be commented and uncommented.

4) Loading:

a) Mechanical (***MECH**) [KM_4]:

Loading option:	2 or 1	(LOP=2) or (LOP=1)
Number of points:	4	(NPT=4)
Time points:	0., 50., 150., 200. sec.	(TI=0.,50.,150.,200.)
Load magnitudes:	0., 40., -40., 0. MPa	(MAG=0.,40.,-40.,0.)
Loading mode:	stress control	(MODE=2,2,2)

For fatigue loading, the loading cycle must be defined in ***MECH**. This cycle must start and end at the same magnitudes. Since this example involves a fully reversed fatigue simulation, the cycle starts at a stress of 0, rises to 40 MPa, then decreases to -40 MPa, and finally returns to 0.

☞ Note: To execute the code with different maximum (and minimum) stress levels (i.e., to generate an S-N curve), the magnitudes of the applied loading must be altered and the code executed repeatedly.

☞ Note: In the case of the laminate, the loading is actual force resultant control. Since the laminate thickness is 1., equivalency exists between the stress magnitudes for the unidirectional composites and the force resultant magnitudes for the laminate.

b) Thermal (***THERM**) [KM_4]:

Number of points:	4	(NPT=4)
Time points:	0., 50., 150., 200. sec.	(TI=0.,50.,150.,200.)
Temperature points:	23., 23., 23., 23.	(TEMP=23.,23.,23.,23.)

As with the mechanical loading, the thermal loading in fatigue damage analysis defines the cycle and must start and end at the same temperature. In this case, the loading cycle does not involve a temperature change.

c) Time integration (***SOLVER**) [KM_4]:

Time integration method:	Forward Euler	(METHOD=1)
Number of points:	4	(NPT=4)
Time points:	0., 50., 150., 200. sec.	(TI=0.,50.,150.,200.)
Time step sizes:	10., 10., 10. sec.	(STP=10.,10.,10.)

5) Damage and Failure:

a) Fatigue Damage Analysis (***DAMAGE**) [KM_5]:

```
MAXNB=100 DINC=0.2 DMAX=1.0 BLOCK=0.,200.
```

Max. no. of load blocks:	100	(MAXNB=100)
Damage increment:	0.2	(DINC=0.2)
Max. damage value:	1.0	(DMAX=1.0)
Load block times:	0., 200. sec.	(BLOCK=0.,200.)

The load block associated with one cycle is specified using BLOCK= to indicate the start and end times of the load block. This allows additional loading to occur before or after the actual cyclic load block if desired (e.g., to incorporate residual stresses). The MAC/GMC 4.0 fatigue analysis applies this load block and then determines the number of cycles of this load block required to cause the local damage increment specified as DINC=. This is a local increment of the damage parameter, *D*, which pertains to a single subcell. Since there can be many subcells, each with its own value of *D*, the code selects a controlling subcell that reaches the damage increment first. The number of cycles (of the specified load block) required to cause the local damage increment is then imposed upon the composite, resulting in a state of damage throughout the composite. Then, the simulated load block is applied again, and a new number of cycles required to increment the local damage (by DINC) is calculated. This process is repeated until the maximum number of load blocks (MAXNB) has been applied or complete failure has occurred. By applying load blocks to increment the damage in this fashion, the stress state in the composite is permitted to redistribute based on the evolving state of damage. In general, a smaller damage increment will cause longer execution times, but also allows a greater degree of load redistribution.

```
NDMAT=2
MAT=1 MOD=2 SU1=3500,91.2,91.2,31.4,134.,134 &
            SU2=2000.,91.2,91.2,31.4,134.,134. &
            N1=1000,1000,1000,1000,1000,1000 &
            N2=300000000,300000000,300000000,300000000,300000000,300000000
```

No. damaging materials:	2	(NDMAT=2)
For Material #1		(MAT=1)
Fatigue model:	Strength reduction model	(MOD=2)
Ultimate stress point 1:	σ_{11} = 3500. MPa	(SU1=3500.,91.2,…)
	σ_{22} = 91.2 MPa	
	σ_{33} = 91.2 MPa	
	σ_{23} = 31.4 MPa	
	σ_{13} = 134. MPa	
	σ_{12} = 134. MPa	
Ultimate stress point 2:	σ_{11} =2000. MPa	(SU1=2000.,91.2,…)
	σ_{22} = 91.2 MPa	
	σ_{33} = 91.2 MPa	
	σ_{23} = 31.4 MPa	
	σ_{13} = 134. MPa	
	σ_{12} = 134. MPa	
Number of cycles point 1:	1000	(N1=1000,1000,…)
Number of cycles point 2:	300,000,000	(N2=300000000,300000000,…)

The ultimate stress points and number of cycles points are listed for the six stress components. In the present example, only the σ_{11} ultimate stress component is changing with number of cycles.

```
MAT=2 MOD=1 ANG=0. BN=0.0 BP=0.0 OMU=1. OMFL=1. OMM=1. ETU=1. &
       ETFL=1. ETM=1. BE=9. A=0.05 SFL=27. XML=150. &
       SU=80.
```

For Material #2		(MAT=2)
Fatigue model:	Stiffness reduction model	(MOD=1)
θ:	0.	(ANG=0.)
b:	0.0	(BN=0.0)
b’:	0.0	(BP=0.0)
ω_u:	1.	(OMU=1.)
ω_{fl}:	1.	(OMFL=1.)
ω_m:	1.	(OMM=1.)
η_u:	1.	(ETU=1.)
η_{fl}:	1.	(ETFL=1.)
η_m:	1.	(ETM=1.)

β:	9.	(BE=9.)
$\hat{a}$:	0.05	(A=9.)
σ_{fl}:	27. MPa	(SFL=27.)
M:	150. MPa	(XML=150.)
σ_u:	80. MPa	(SU=80.)

For the meaning of all stiffness reduction fatigue damage model parameters, see the MAC/GMC 4.0 Theory Manual Section 5.3. For an isotropic material such as epoxy, the last 5 parameters listed are all that must be selected.

b) Subcell static failure analysis (***FAILURE_SUBCELL**) [KM_5]:

Number of materials:	2	(NMAT=1)
Material #1		(MAT=1)
Number of criteria:	1	(NCRIT=1)
Criterion #1:	Maximum stress criterion	(CRIT=1)
Failure stresses:	σ_{11} = 3500. MPa	(X11=3500.)
	σ_{22} = 91.2 MPa	(X22=91.2)
	σ_{33} = 91.2 MPa	(X33=91.2)
	σ_{23} = 31.4 MPa	(X23=31.4)
	σ_{13} = 134. MPa	(X13=134.)
	σ_{12} = 134. MPa	(X12=134.)
Compression flag:	Compressive strengths same as tensile	(COMPR=SAM)
Material #2		(MAT=2)
Number of criteria:	1	(NCRIT=1)
Criterion #1:	Maximum stress criterion	(CRIT=1)
Failure stresses:	σ_{11} = 80. MPa	(X11=80.)
	σ_{22} = 80. MPa	(X22=80.)
	σ_{33} = 80. MPa	(X33=80.)
	σ_{23} = 40. MPa	(X23=40.)
	σ_{13} = 40. MPa	(X13=40.)
	σ_{12} = 40. MPa	(X12=40.)
Compression flag:	Compressive strengths same as tensile	(COMPR=SAM)

☞ Note: In the case of fatigue damage analysis, ACTION is not needed because the fatigue analysis must continue until overall failure or the maximum number of load blocks is applied. If ACTION is specified in this case, it will be ignored by the code.

c) RUC static failure analysis (***FAILURE_CELL**) [KM_5]:

Number of criteria:	1	(NCRIT=1)
Criterion #1:	Maximum strain criterion	(CRIT=2)
Failure strains:	ε_{11} = 0.05	(X11=0.05)
	ε_{22} = 0.05	(X22=0.05)
	ε_{33} = 0.05	(X33=0.05)
	ε_{23} = 0.05	(X23=0.05)

	$\varepsilon_{13} = 0.05$	(X13=0.05)
	$\varepsilon_{12} = 0.05$	(X12=0.05)
Compression flag:	Compressive strains same as tensile	(COMPR=SAM)

In the present example, the RUC static failure analysis option is employed to limit the amount of strain permitted for an RUC. This allows the code to treat the RUC as failed if damage has occurred such that the stiffness of the RUC is very low and a large strain results. Limiting the strain in this matter can also prevent numerical overflow when the loading is in stress control, as in the present example. The RUC level static failure capabilities function similarly to the subcell level static failure capabilities. The code simply employs the global quantities rather than the local quantities when evaluating the appropriate failure criteria.

6) Output:

a) Output file print level (***PRINT**) [KM_6]:

Print level:	3	(NPL=3)

b) x-y plots (***XYPLOT**):

Frequency:	1	(FREQ=1)
Number of laminate plots:	1	(LAMINATE=1)
Laminate plot names:	example_5d	(NAME=example_5d)
Laminate x-y quantities:	ε^0_{xx}, N_{xx}	(X=1 Y=10)
Number of macro plots:	2	(MACRO=2)
Macro plot names:	example_5d_11	(NAME=example_5d_11)
	example_5d_22	(NAME=example_5d_22)
Macro plot x-y quantities:	ε_{11}, σ_{11}	(X=1 Y=7)
	ε_{22}, σ_{22}	(X=2 Y=8)
Number of micro plots:	0	(MICRO=0)

☞ Note: In this example, the lines from the input file associated with the laminate plots should be commented for the RUC analyses.

7) End of file keyword: (***END**)

Results

Results for the fatigue damage analysis are written not only to the MAC/GMC 4.0 output file, but also to a damage file. This damage file is given the same name as the output file, with `_dam.data` appended. Thus, in the present example, the damage file is named "`example_5d_dam.data`". This file includes a summary of the damage calculations and state of damage after each applied load block, as well as the total number of cycles to failure. A portion of this file is shown below:

```
Completed Applied Load Block Number   3   (Loading Increment Number     60 )

 ----------------------------------------------------------------------------

    A) Cycles Required to Incur a Damage Increment of DINC = 0.2000
       Controlling subcell --->   46
       Number of Cycles =>        1460.389
       Number of Cycles =>        1460.000 (rounded)
```

```
 -----------------------------------------------

  B) Current TOTAL number of cycles --->      7530637
     (after applied load cycle    3,   D = 0.5999 )

 -----------------------------------------------

  C) Current Damage in each subcell (after      7530637 cycles):

      Subcell         NF*             D
      -------       -------        -------
         1          - inf -         0.0000
         2          - inf -         0.0000
         3         0.2876E+09       0.0000
         4          1486.0947       0.5993
         5         0.2876E+09       0.0000
         6          - inf -         0.0000
         7          - inf -         0.0000
         8          - inf -         0.0000
```

.
. → **Lines Omitted**
.

```
      *NOTE: NF = Remaining life assuming no further stress redistribution
```

.
. → **Lines Omitted**
.

```
  ********************************************
  *        ALL SUBCELLS HAVE FAILED          *
  *   TOTAL NUMBER OF CYCLES =    7530637.0   *
  *                                          *
  ********************************************
```

Results from the x-y plot file are shown in Figure 5.7 for a transverse fatigue analysis of the unidirectional 60% graphite/epoxy composite. In this case, it was determined that 7,517,751 cycles of load block #1 were required to achieve a damage level of 0.2 in a subcell. Then, load block #2 was applied, and, as shown in Figure 5.7, the transverse composite response becomes more compliant due to the damage state caused by the 7,517,751 cycles of load block #1. Only 11,426 cycles of load block #2 are required to cause a local damage increment of 0.2, resulting in a total number of cycles of 7,529,177. When load block #3 is applied, the transverse composite response is even more compliant due to the higher level of damage throughout the composite. Only 1,460 cycles of load block #3 are required to cause an additional local damage increment of 0.2. During applied load block #4 local failures occur that cause the non-linearity evident in Figure 5.7. Then, when the code attempts to apply load block #5, additional failures occur that lead to the complete failure of the composite. That is, after application of load block #4, the code cannot withstand any additional cycles. The predicted life of the composite is 7,530,637 cycles.

By altering the applied stress magnitudes in **`*MECH`**, a transverse S-N curve can be predicted for the graphite/epoxy composite. Similarly, by altering the loading option, a longitudinal S-N curve for the composite can be predicted. Finally, switching the simulation to a laminate fatigue analysis (commenting **`*RUC`** and uncommenting **`*LAMINATE`** in the input file), an S-N curve for the quasi-isotropic laminate can be predicted. All three of these predicted S-N curves are plotted in Figure 5.8, whereas, for additional detail, each S-N curve is plotted separately in Figure 5.9, Figure 5.10, and Figure 5.11. As one would expect, the S-N curve for the longitudinal composite is highest due to the strong and stiff fibers oriented along the loading direction. The transverse S-N curve is lowest since the composite's transverse response is matrix dominated, and the laminate S-N curve is intermediate.

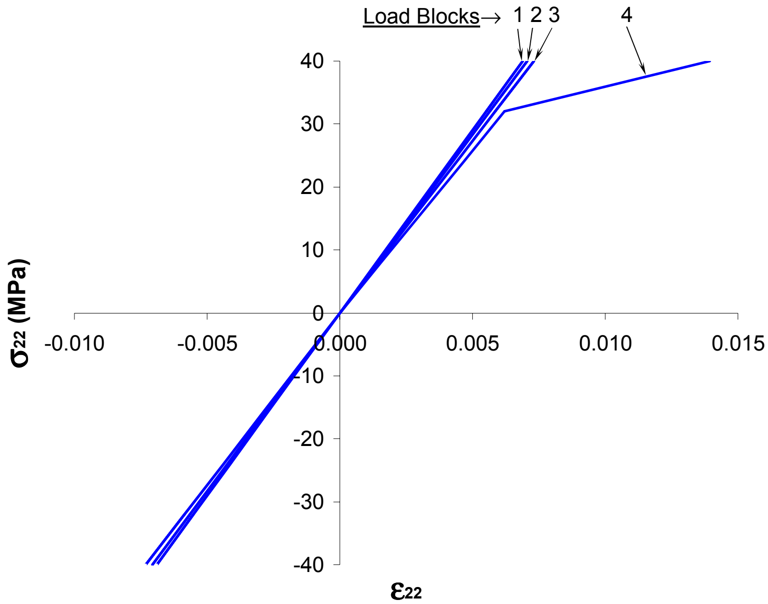

Figure 5.7 Example 5d: Transverse stress-strain response for a 60% graphite/epoxy composite at room temperature for the four applied load blocks prior to failure.

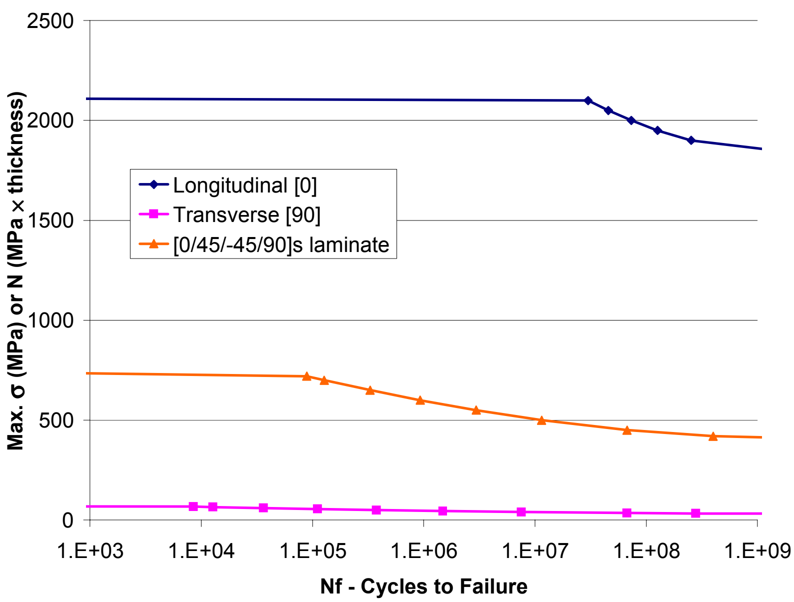

Figure 5.8: Example 5d: Predicted S-N curves for 60% graphite/epoxy at room temperature.

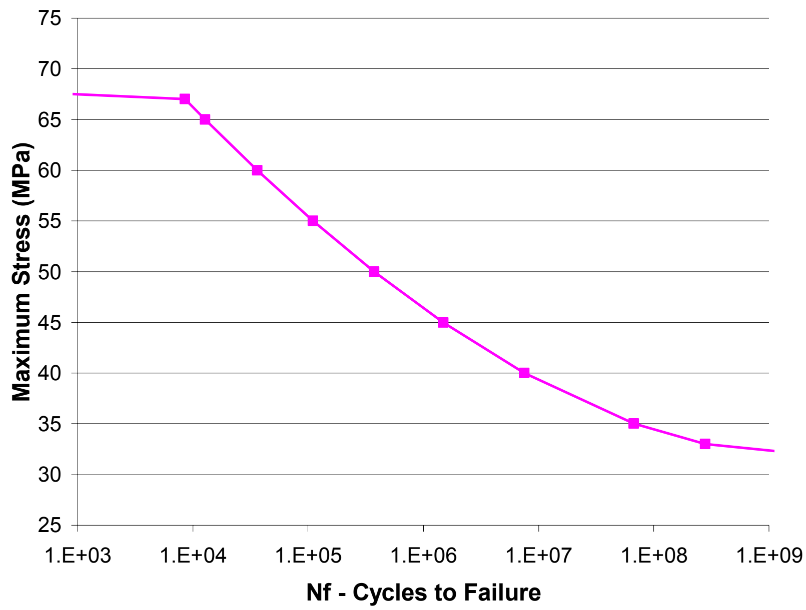

Figure 5.9: Example 5d: Predicted transverse S-N curve for 60% graphite/epoxy at room temperature.

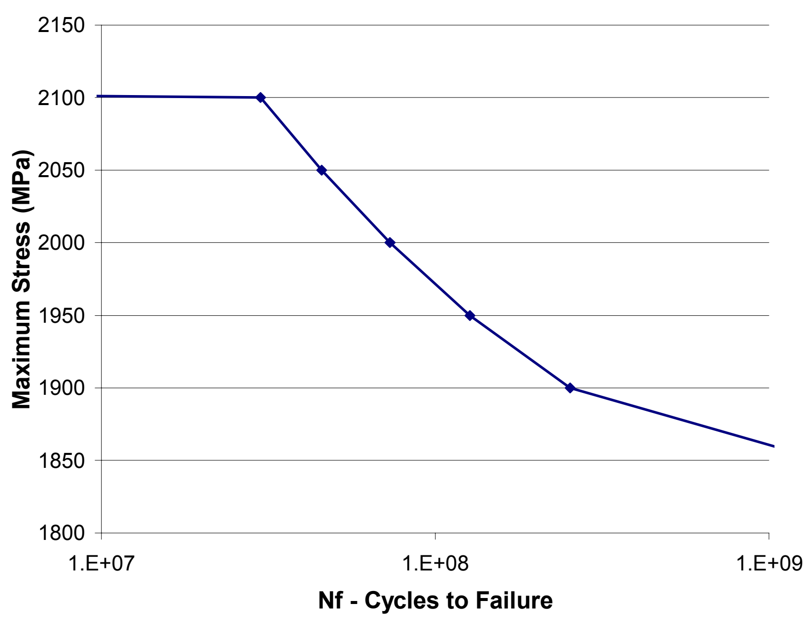

Figure 5.10 Example 5d: Predicted longitudinal S-N curve for 60% graphite/epoxy at room temperature.

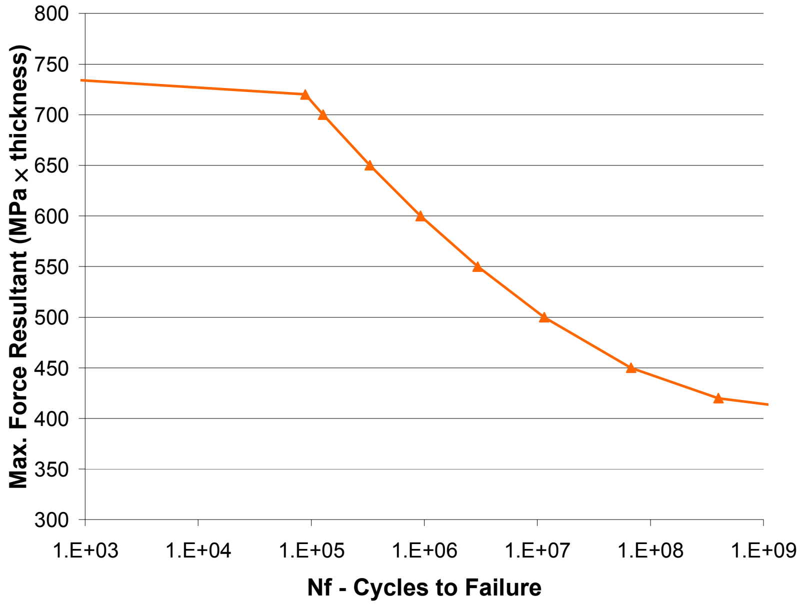

Figure 5.11 Example 5d: Predicted S-N curve for a quasi-isotropic [0°/45°/-45°/90°]$_s$ 60% graphite/epoxy laminate.