Example 4e: General loading Option for Laminates

This problem exercises the general loading option for a laminate. As in the previous example problem, the loading associated with the laminate involves force and moment resultants along with midplane strains and curvatures. Now, however, all six components applied to the laminate are specified via the LOP=99 loading option. In particular, an asymmetric elastic graphite/epoxy laminate is considered with temperature-dependent user-defined constituent properties. The laminate is subjected to a stress-free cool-down from 150 °C, which is intended to simulate the cure temperature. During the cool-down, the asymmetric laminate experiences bending (non-zero curvatures), and the subsequent mechanical loading eliminates this bending by returning the curvatures to zero. Thus, at the completion of the applied loading, the code has determined the moment resultants required to flatten the laminate after the thermally induced curvature.

MAC/GMC Input File: **example_4e.mac**

```
MAC/GMC 4.0 Example 4e - General loading option for laminates
*CONSTITUENTS
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=388.2E9,390.E9
  ET=7.6E9,7.6E9
  NUA=0.41,0.41
  NUT=0.45,0.45
  GA=14.9E9,15.1E9
  ALPA=-0.68E-6,-0.45E-6
  ALPT=9.74E-6,10.34E-6
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=3.45E9,3.10E9
  ET=3.45E9,3.10E9
  NUA=0.35,0.35
  NUT=0.35,0.35
  GA=1.278E9,1.148E9
  ALPA=45.E-6,55.E-6
  ALPT=45.E-6,55.E-6
*LAMINATE
  NLY=4
  LY=1 MOD=2 THK=0.25 ANG=90  ARCHID=6 R=1. VF=0.65 F=1 M=2
  LY=2 MOD=2 THK=0.25 ANG=60  ARCHID=6 R=1. VF=0.65 F=1 M=2
  LY=3 MOD=2 THK=0.25 ANG=-60 ARCHID=6 R=1. VF=0.65 F=1 M=2
  LY=4 MOD=2 THK=0.25 ANG=0   ARCHID=6 R=1. VF=0.65 F=1 M=2
*MECH
  LOP=99
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,2
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,2
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,2
```

```
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,1
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,1
  NPT=3 TI=0.,1.,2. MAG=0.,0.,0. MODE=2,1
*THERM
  NPT=3 TI=0.,1.,2. TEMP=150.,23.,23.
*SOLVER
  METHOD=1 NPT=3 TI=0.,2. STP=1.
*PRINT
  NPL=0
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

Number of materials:	2	(NMATS=2)
Constitutive models:	Elastic	(CMOD=6)
Materials:	User-defined (Graphite)	(MATID=U)
	User-defined (Epoxy)	(MATID=U)
Material property source:	Read from input file	(MATDB=1)
Material properties:	See Table 4.1	

Table 4.1 Constituent material elastic properties for Example 4e.

	Temp (°C)	E_A (GPa)	E_T (GPa)	ν_A	ν_T	G_A (GPa)	α_A (10^{-6}/ °C)	α_T (10^{-6}/ °C)
Graphite	23.	388.2	7.6	0.41	0.45	14.9	-0.68	9.74
	150.	390.	7.6	0.41	0.45	15.1	-0.45	10.34
Epoxy	23.	3.45	3.45	0.35	0.35	1.278	45.	45.
	150.	3.10	3.10	0.35	0.35	1.148	55.	55.

3) Analysis type (***LAMINATE**) → Laminate Analysis [KM_3]:

Number of layers: 4 (NLY=4)

Layer	**Analysis Model**	**Thickness**	**Fiber Angle**	**Architecture**	**Volume fraction**	**Aspect ratio**	**Fiber material**	**Matrix material**
(LY=)	(MOD)	(THK)	(ANG)	(ARCHID)	(VF)	(R)	(F)	(M)
1	GMC-2D	0.25	90°	7×7 circle, rect. pack	0.65	1.	graphite	epoxy
2	GMC-2D	0.25	60°	7×7 circle, rect. pack	0.65	1.	graphite	epoxy
3	GMC-2D	0.25	-60°	7×7 circle, rect. pack	0.65	1.	graphite	epoxy
4	GMC-2D	0.25	0°	7×7 circle, rect. pack	0.65	1.	graphite	epoxy

4) Loading:
a) Mechanical (**`*MECH`**) [KM_4]:
Loading option: general loading (`LOP=99`)

<u>Component #1</u> (ε^0_{xx} or N_{xx})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	force resultant	force resultant	

<u>Component #2</u> (ε^0_{yy} or N_{yy})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	force resultant	force resultant	

<u>Component #3</u> (γ^0_{xy} or N_{xy})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	force resultant	force resultant	

<u>Component #4</u> (κ_{xx} or M_{xx})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	moment resultant	curvature	

<u>Component #5</u> (κ_{yy} or M_{yy})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	moment resultant	curvature	

<u>Component #6</u> (κ_{xy} or M_{xy})
Number of points: 3 (`NPT=3`)

Times (`TI=`) (sec.)	0.	1.	2.
Magnitudes (`MAG=`)	0.	0.	0.
Control (`MODE=`)	moment resultant	curvature	

b) Thermal (**`*THERM`**) [KM_4]:

Number of points:	3	`(NPT=3)`
Time points:	0., 1., 2. sec.	`(TI=0.,1.,2.)`
Temperature points:	150., 23., 23. °C	`(TEMP=150.,23.,23.)`

c) Time integration (**`*SOLVER`**) [KM_4]:

Time integration method:	Forward Euler	`(METHOD=1)`
Number of points:	3	`(NPT=3)`
Time points:	0., 1., 2. sec.	`(TI=0.,1.,2.)`
Time step sizes:	1., 1. sec.	`(STP=1.,1.)`

☞ Note: Since this example problem is linearly elastic and the temperature-dependent constituent material properties are linear, the problem solution is independent of the number of time steps taken through the loading history. Therefore the minimum number of time steps (2) has been employed.

5) Damage and Failure: None

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

Print level: 6 `(NPL=6)`

b) x-y plots (**`*XYPLOT`**): None

7) End of file keyword: (**`*END`**)

Results

The results below are taken from the MAC/GMC 4.0 output file for this example.

```
    1 TIME:  1.0000D+00      TEMP:  2.3000D+01      TSTEP:  1.0000D+00
----------------------------------------------------------------------------------------------------
     FORCE(N), MOMENT(M):  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
       STRAIN, CURVATURE: -1.2843D-03  -5.4894D-05   1.3757D-04   3.3269D-03  -4.3171D-04  -3.6875D-03
          INELASTIC N, M:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
            THERMAL N, M: -1.6159D+07  -1.0899D+07  -1.0789D-03   1.9726D+06  -1.9726D+06  -5.6944D+05
     OUT-OF-PLANE STRAIN: -5.2503D-03   0.0000D+00   0.0000D+00

    2 TIME:  2.0000D+00      TEMP:  2.3000D+01      TSTEP:  1.0000D+00
----------------------------------------------------------------------------------------------------
     FORCE(N), MOMENT(M):  0.0000D+00   0.0000D+00   0.0000D+00  -6.5415D+06   2.9887D+06   1.1301D+06
       STRAIN, CURVATURE: -1.9617D-04  -4.3628D-05  -6.2573D-14   0.0000D+00   0.0000D+00   0.0000D+00
          INELASTIC N, M:  0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00   0.0000D+00
            THERMAL N, M: -1.6159D+07  -1.0899D+07  -1.0789D-03   1.9726D+06  -1.9726D+06  -5.6944D+05
     OUT-OF-PLANE STRAIN: -5.7715D-03   0.0000D+00   0.0000D+00
```

Upon completion of the stress-free cool down of the laminate (at time = 1. sec.), the laminate is bent (as indicated by the non-zero curvatures). Then, upon completion of the simulated mechanical loading (at time = 2. sec.), the laminate is no longer bent (as indicated by the zero curvatures). The moment resultants required to eliminate the laminate's bending are: $M_{xx} = -6.54$ MPa × thickness^2, $M_{yy} = 2.99$ MPa × thickness^2, $M_{xy} = 1.13$ MPa × thickness^2, where thickness is the total thickness of the laminate (e.g., if the laminate were 1 mm thick, the required

$M_{xx} = -6.54 \times 10^6 \ \frac{\text{N}}{\text{m}^2} \times (0.001 \ \text{m})^2 = -6.54 \ \text{N}$). Recall that a moment resultant is a moment (with units force $\times$ length) per unit width of the laminate, yielding units of force.