Example 1a: Effective Properties of a Composite

This example problem determines the effective thermal and mechanical properties of a continuous graphite fiber/epoxy matrix composite material. The simplest repeating unit cell (RUC) available within MAC/GMC 4.0 that can represent the architecture of this composite, namely a 2×2 RUC (see Figure 1.1), is employed. As Figure 1.1 shows, the doubly periodic RUC consists of four subcells in the x_2-x_3 plane, one of which represents the fiber and three of which represent the matrix. The term "doubly periodic" indicates that the RUC repeats infinitely in the two in-plane (x_2-x_3) directions and is infinite in the out-of-plane (continuously reinforced, x_1) direction. The RUC thus represents a continuum (as opposed to a structure with boundaries). Based on the properties and arrangement of the fiber and matrix constituents, MAC/GMC 4.0 uses the doubly periodic generalized method of cells (GMC) theory to homogenize the composite material and determine the effective (anisotropic) properties of this continuously reinforced homogenized material. The constituent material properties in this problem are temperature-independent and are read from the MAC/GMC input file.

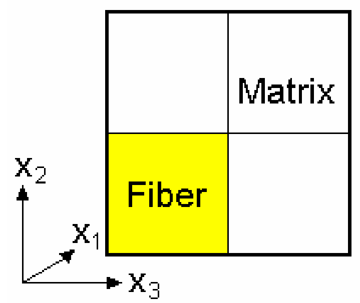

Figure 1.1 MAC/GMC 4.0 2×2 doubly periodic repeating unit cell.

MAC/GMC Input File: **example_1a.mac**

```
MAC/GMC 4.0 Example 1a - graphite/epoxy effective properties
*CONDUCTIVITY
 NTEMP=1 TEMP=21.
*CONSTITUENTS
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1 &
   EL=388.2E9,7.6E9,0.41,0.45,14.9E9,-0.68E-6,9.74E-6
   K=500.,10.
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1 &
   EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45E-6
   K=0.19,0.19
*RUC
  MOD=2 ARCHID=1 VF=0.65 F=1 M=2
*PRINT
  NPL=-1
*END
```

☞ Note: The lines of the input file starting with the "#" character are comments and thus ignored by the code.

Annotated Input Data

1) Flags:

a) Determine effective thermal conductivity properties (**`*CONDUCTIVITY`**) [KM_1]:

```
NTEMP=1 TEMP=21.
```

Number of conductivity temps:	1	(`NTEMP=1`)
Conductivity temps:	21.	(`TEMP=21.`)

`*CONDUCTIVITY` is a flag-type keyword that indicates to MAC/GMC 4.0 that effective thermal conductivities should be calculated. The thermal conductivity calculation can be performed at any desired temperature (specified by `NTEMP=` and `TEMP=`). This feature is useful when the constituent properties (specified under **`*CONSTITUENTS`**) are temperature-dependent. In this example problem, the material properties are temperature-independent, so even if additional conductivity temperatures were specified, the calculated effective thermal conductivities would be the same for each temperature. Note that, in the presence of the **`*CONDUCTIVITY`** keyword, thermal conductivities for the constituent materials must be specified under **`*CONSTITUENTS`**.

2) Constituent materials (**`*CONSTITUENTS`**) [KM_2]:

```
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1 &
   EL=388.2E9,7.6E9,0.41,0.45,14.9E9,-0.68E-6,9.74E-6
   K=500.,10.
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1 &
   EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45.E-6
   K=0.19,0.19
```

Number of materials:	2	(`NMATS=2`)
Constitutive models:	Elastic	(`CMOD=6`)
Materials:	User-defined (Graphite)	(`MATID=U`)
	User-defined (Epoxy)	(`MATID=U`)
Material property source:	Read from input file	(`MATDB=1`)
Material properties:	See Table 1.1	(`EL=`... and `K=`...)

Table 1.1 Constituent material properties for example 1a.

	E_A (GPa)	E_T (GPa)	ν_A	ν_T	G_A (GPa)	α_A (10^{-6}/ °C)	α_T (10^{-6}/ °C)	κ_A (W/mK)	κ_T (W/mK)
Graphite	388.2	7.6	0.41	0.45	14.9	-0.68	9.74	500.	10.
Epoxy	3.45	3.45	0.35	0.35	1.278	45.	45.	0.19	0.19

This example problem employs user-defined material properties. In the present case of elastic material constitutive behavior (as specified by `CMOD=6`), the temperature-independent elastic properties and coefficients of thermal expansion (CTEs) are simply listed as `EL=`E_A, E_T, ν_A, ν_T, G_A, α_A, α_T (axial and transverse elastic modulus, axial and transverse Poisson ratio, axial shear modulus, and axial and transverse CTE, respectively). The temperature-independent thermal conductivities are specified on a separate line as K=κ_A, κ_T (axial and transverse thermal conductivity, respectively). Note that, while the K=... specification is required when the **`*CONDUCTIVITY`** keyword is present, in the absence of **`*CONDUCTIVITY`**, K=... should be omitted. In addition, the employed elastic constitutive model (`CMOD=6`) allows for transversely isotropic material properties. That is, it allows for specification of different properties in the axial and transverse direction (as specified for the graphite fiber, for example). Care should be exercised, however, as `CMOD=6` requires that the axial direction be associated with the x_1-axis (see Figure 1.1). For transversely isotropic elastic constituent behavior with axes other than the x_1-axis, constitutive model 9 (`CMOD=9`) can be employed (see Keywords Manual Section 2). The isotropic simplification of the employed elastic model (`CMOD=6`) is obtained by setting $E_A = E_T = E$, $\nu_A = \nu_T = \nu$, $G_A = E/2(1+\nu) = G$, $\alpha_A = \alpha_T = \alpha$, and $\kappa_A = \kappa_T$.

In addition to direct specification of the constituent material properties within the MAC/GMC 4.0 input file, several other options exist for material property specification. These are: material properties taken from the code's internal database, material properties read from an external database, and material properties determined from the user-defined subroutine (`usrfun.F90`). Also, for different constitutive models (specified by `CMOD=`), and for temperature-dependent properties, the input requirements are different. These options and requirements are described in Section 2 of the MAC/GMC 4.0 Keywords Manual, and example problems involving these options and requirements are presented in Section 2 of this Example Problem Manual.

The units of the material properties employed by MAC/GMC 4.0 are arbitrary. However, the units must be consistent in order for the MAC/GMC 4.0 results to be meaningful. It is the user's responsibility to ensure that the constituent material property units are consistent. This is particularly important when employing constituent materials from the internal material property database. For more information on constitutive material property units, see the MAC/GMC 4.0 Keywords Manual Section 2.

3) Analysis type (**`*RUC`**) → Repeating Unit Cell Analysis [KM_3]:

```
MOD=2 ARCHID=1 VF=0.65 F=1 M=2
```

Analysis model:	Doubly periodic GMC	(`MOD=2`)
RUC architecture:	square fiber, square pack	(`ARCHID=1`)
Fiber volume fraction:	0.65	(`VF=0.65`)
Material assignment:	graphite fiber	(`F=1`)
	epoxy matrix	(`M=2`)

By including **`*RUC`** within the input file, repeating unit cell analysis, which represents a continuum has been selected. In this example, a repeating unit cell architecture is selected from the MAC/GMC internal library by specifying `ARCHID=1`. This RUC, which is the simplest representation of a continuous fiber composite, is shown in Figure 1.1. The additional information required for this RUC architecture is the fiber volume fraction and which of the two constituent materials occupy the subcells associated with the fiber (`F=`) and the matrix (`M=`). Based on the specified fiber volume fraction, MAC/GMC 4.0 determines the dimensions of each subcell. Then, given the information on which materials occupy which subcells, the code has obtained all of the data (the constituent properties and their arrangement) required to homogenize the composite and determine its effective properties via the generalized method of cells theory.

☞ Note: While only `VF=`, `F=`, and `M=` are required for `ARCHID=1`, other RUC architectures contained within the MAC/GMC 4.0 RUC architecture library require unique input data. Example problems dealing with other RUC architectures are presented within Section 3 of this Example Manual. For additional information on RUC architectures, see the MAC/GMC 4.0 Keywords Manual Section 3.

4) Loading: None

As stated earlier, this example problem involves determining effective properties only. Therefore, no simulated thermal or mechanical loading need be specified. See Example Problem 1c for an introduction on specifying applied loading histories.

5) Damage and Failure: None

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

```
NPL=-1
```

Print level: -1 (effective properties only) (`NPL=-1`)

The print level generally indicates how much information will be written to the output file. A level of 0 prints the least amount of information while a level of 10 prints the most. In addition, a special case of `NPL=-1` indicates that the code should execute only to determine effective properties (as opposed to determining the response to simulated applied loading). See the MAC/GMC 4.0 Keywords Manual Section 6 for details on the various print levels.

☞ Note: If `NPL≠-1` at least one of the keywords **`*MECH`** and **`*THERM`** must be present in the input file.

b) x-y plots (**`*XYPLOT`**): None

Because no simulated mechanical loading is applied in the present example, no x-y plot data will be written to output files. See Example Problem 1c for an introduction to generating x-y plots.

7) End of file keyword: (**`*END`**)

The **`*END`** end of file keyword must be included in all MAC/GMC 4.0 input files.

Results

In this example problem, since MAC/GMC 4.0 is employed only to determine the effective properties of the graphite/epoxy composite, all results are contained within the output file. Recall that (as described in the Keywords Manual), if the code is executed using the command:

```
mac4 example_1a
```

then the relevant output is written to the ASCII file `example_1a.out`. The entirety of this output file is provided in the appendix, and the results given below were taken directly from this output file.

The results include the effective thermal conductivities of the 0.65 fiber volume fraction graphite/epoxy composite, the effective stiffness matrix (CG), the inverse of the effective compliance matrix (CI), the effective engineering moduli (i.e., effective elastic properties), and the effective coefficients of thermal expansion. Recall from Figure 1.1 that the axial fiber direction is associated with the x_1-axis, while the x_2 and x_3 directions are transverse to the continuous fiber. The high thermal conductivity, high stiffness, and low CTE associated with the axial direction of the fiber constituent are evident in the effective properties of the composite associated with the x_1 direction. Transverse to the fiber direction, on the other hand, the thermal conductivity and stiffness are low while the CTE is high.

```
-------------------------------------------------
 Effective Thermal Conductivities

  At Temperature =   21.0
    K11 =     325.0665
    K22 =       0.7694
    K33 =       0.7694

-------------------------------------------------


-------------------- EFFECTIVE PROPERTIES AT TEMPERATURE =    21.00 ---------------------------



 CG - Effective/Macro Stiffness Matrix

    0.2571E+12    0.4500E+10    0.4500E+10
    0.4500E+10    0.7828E+10    0.3707E+10
    0.4500E+10    0.3707E+10    0.7828E+10
                                             0.1916E+10
                                                           0.4166E+10
                                                                         0.4166E+10

 CI - Effective/Macro Compliance Matrix

    0.3944E-11   -0.1539E-11   -0.1539E-11
   -0.1539E-11    0.1653E-09   -0.7738E-10
   -0.1539E-11   -0.7738E-10    0.1653E-09
                                             0.5219E-09
                                                           0.2400E-09
                                                                         0.2400E-09

 Effective Engineering Moduli

     E11S= 0.2535E+12
     N12S= 0.3901
     E22S= 0.6050E+10
     N23S= 0.4682
     E33S= 0.6050E+10
     G23S= 0.1916E+10
     G13S= 0.4166E+10
     G12S= 0.4166E+10

 Effective Thermal Expansion Coefficients

   -0.4724E-06    0.2663E-04    0.2663E-04


-----------------------------------------------------------------------------------------------
```