Example 5c: Laminate Static Failure Analysis

In this example problem, a 60% fiber volume fraction graphite/epoxy composite is again considered, as in Examples 5a and 5b. However, now a quasi-isotropic $[0°/45°/-45°/90°]_s$ laminate is the subject of static failure analysis utilizing both the subcell and RUC static failure analysis options. Biaxial loading (N_{xx}-N_{yy}) is applied to the laminate, and by altering the load magnitudes applied in the two directions, a plot of the predicted laminate "failure surface" is generated. This "failure surface" is the locus of points in the plane of the applied biaxial stress loading at which failure occurs. Unlike the MAC/GMC 4.0 yield surface capabilities, this failure surface analysis procedure is not automated at this time.

MAC/GMC Input File: **example_5c.mac**

```
MAC/GMC 4.0 Example 5c - Laminate static failure analysis
*CONSTITUENTS
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=388.2E3,390.E3
  ET=7.6E3,7.6E3
  NUA=0.41,0.41
  NUT=0.45,0.45
  GA=14.9E3,15.1E3
  ALPA=-0.68E-6,-0.45E-6
  ALPT=9.74E-6,10.34E-6
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1
  NTP=2
  TEM=23.,150.
  EA=3.45E3,3.10E3
  ET=3.45E3,3.10E3
  NUA=0.35,0.35
  NUT=0.35,0.35
  GA=1.278E3,1.148E3
  ALPA=45.E-6,55.E-6
  ALPT=45.E-6,55.E-6
*LAMINATE
  NLY=7
  LY=1 THK=0.125 ANG=0   MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=2 THK=0.125 ANG=45  MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=3 THK=0.125 ANG=-45 MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=4 THK=0.250 ANG=90  MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=5 THK=0.125 ANG=-45 MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=6 THK=0.125 ANG=45  MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
  LY=7 THK=0.125 ANG=0   MOD=2 ARCHID=13 VF=0.60 R=1. F=1 M=2
*MECH
  LOP=99
  NPT=2 TI=0.,100. MAG=0.,2000 MODE=2
  NPT=2 TI=0.,100. MAG=0.,2000 MODE=2
  NPT=2 TI=0.,100. MAG=0.,0. MODE=2
```

```
  NPT=2 TI=0.,100. MAG=0.,0. MODE=2
  NPT=2 TI=0.,100. MAG=0.,0. MODE=2
  NPT=2 TI=0.,100. MAG=0.,0. MODE=2
*THERM
  NPT=2 TI=0.,100. TEMP=23.,23.
*SOLVER
  METHOD=1 NPT=2 TI=0.,100. STP=0.1
*FAILURE_SUBCELL
  NMAT=2
  MAT=1 NCRIT=1
   CRIT=3 X11=3500. X22=91.2  X33=91.2  X23=31.4  X13=134.  X12=134.  &
    ACTION=-1
  MAT=2 NCRIT=1
   CRIT=3 X11=80.   X22=80.   X33=80.   X23=40.   X13=40.   X12=40.   &
    ACTION=-1
#*FAILURE_CELL
#  NCRIT=1
# -- Data from 1st subcell allowables (Example 5a)
#  CRIT=3 X11=2112. X22=68.08  X33=68.08  X23=31.4  X13=24.89 X12=24.89  &
#    ACTION=-1
# -- Data from avg material allowables (Example 5a)
#  CRIT=3 X11=2112. X22=84.16  X33=84.16  X23=31.4  X13=57.64 X12=57.64  &
#    ACTION=-1
*PRINT
  NPL=6
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

Number of materials:	2	(NMATS=2)
Constitutive models:	Elastic	(CMOD=6)
Materials:	User-defined (Graphite)	(MATID=U)
	User-defined (Epoxy)	(MATID=U)
Material property source:	Read from input file	(MATDB=1)
Material properties:	See Table 4.1	

3) Analysis type (***LAMINATE**) → Laminate Analysis [KM_3]:

Number of layers:	7	(NLY=7)

Layer	Analysis Model	Thickness	Fiber Angle	Architecture	Volume fraction	Aspect ratio	Fiber material	Matrix material
(LY=)	(MOD)	(THK)	(ANG)	(ARCHID)	(VF)	(R)	(F)	(M)
1	GMC-2D	0.125	0°	26×26 circle rect. pack	0.60	1.	graphite	epoxy
2	GMC-2D	0.125	45°	26×26 circle rect. pack	0.60	1.	graphite	epoxy
3	GMC-2D	0.125	-45°	26×26 circle rect. pack	0.60	1.	graphite	epoxy

4	GMC-2D	0.25	90°	26×26 circle rect. pack	0.60	1.	graphite	epoxy
5	GMC-2D	0.125	-45°	26×26 circle rect. pack	0.60	1.	graphite	epoxy
6	GMC-2D	0.125	45°	26×26 circle rect. pack	0.60	1.	graphite	epoxy
7	GMC-2D	0.125	0°	26×26 circle rect. pack	0.60	1.	graphite	epoxy

4) Loading:

a) Mechanical (**`*MECH`**) [KM_4]:

Loading option: general loading (`LOP=99`)

<u>Component #1</u> (ε_{xx}^{0} or N_{xx})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	2000. MPa × thickness
Control (`MODE=`)	force resultant	

<u>Component #2</u> (ε_{yy}^{0} or N_{yy})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	2000. MPa × thickness
Control (`MODE=`)	force resultant	

<u>Component #3</u> (γ_{xy}^{0} or N_{xy})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	force resultant	

<u>Component #4</u> (κ_{xx} or M_{xx})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	moment resultant	

<u>Component #5</u> (κ_{yy} or M_{yy})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	moment resultant	

Component #6 (κ_{xy} or M_{xy})

Number of points: 2 (`NPT=2`)

Times (`TI=`) (sec.)	0.	100.
Magnitudes (`MAG=`)	0.	0.
Control (`MODE=`)	moment resultant	

☞ Note: To generate the "failure surface" shown in the results for this example problem, the magnitudes of the component #1 and #2 applied loads must be adjusted accordingly. For example, applying N_{xx}=0 and N_{yy}=2000 MPa × thickness will allow the determination of the failure points on the positive N_{yy} -axis in Figure 5.4.

b) Thermal (**`*THERM`**) [KM_4]:

Number of points:	2	(`NPT=2`)
Time points:	0., 100. sec.	(`TI=0.,100.`)
Temperature points:	23., 23.	(`TEMP=23.,23.`)

c) Time integration (**`*SOLVER`**) [KM_4]:

Time integration method:	Forward Euler	(`METHOD=1`)
Number of points:	2	(`NPT=2`)
Time points:	0., 100. sec.	(`TI=0.,100.`)
Time step size:	0.1 sec.	(`STP=0.1`)

5) Damage and Failure:

a) Subcell static failure analysis (**`*FAILURE_SUBCELL`**) [KM_5]:

Number of materials:	2	(`NMAT=2`)
Material #1		(`MAT=1`)
Number of criteria:	1	(`NCRIT=1`)
Criterion #1:	Tsai-Hill criterion	(`CRIT=3`)
Failure stresses:	σ_{11} = 3500. MPa	(`X11=3500.`)
	σ_{22} = 91.2 MPa	(`X22=91.2`)
	σ_{33} = 91.2 MPa	(`X33=91.2`)
	σ_{23} = 31.4 MPa	(`X23=31.4`)
	σ_{13} = 134. MPa	(`X13=134.`)
	σ_{12} = 134. MPa	(`X12=134.`)
Action to take upon failure:	Stop execution	(`ACTION=-1`)
Material #2		(`MAT=2`)
Number of criteria:	1	(`NCRIT=1`)
Criterion #1:	Tsai-Hill criterion	(`CRIT=3`)
Failure stresses:	σ_{11} = 80. MPa	(`X11=80.`)
	σ_{22} = 80. MPa	(`X22=80.`)

	σ_{33} = 80. MPa	(`X33=80.`)
	σ_{23} = 40. MPa	(`X23=40.`)
	σ_{13} = 40. MPa	(`X13=40.`)
	σ_{12} = 40. MPa	(`X12=40.`)
Action to take upon failure:	Stop execution	(`ACTION=-1`)

b) RUC static failure analysis (**`*FAILURE_CELL`**) [KM_5]:

Number of criteria:	1	(`NCRIT=1`)
Criterion #1:	Tsai-Hill criterion	(`CRIT=3`)

Data From 1st Subcell Allowables for Composite

Failure stresses:	σ_{11} = 2112. MPa	(`X11=2112.`)
	σ_{22} = 68.08 MPa	(`X22=68.08`)
	σ_{33} = 68.08 MPa	(`X33=68.08`)
	σ_{23} = 31.4 MPa	(`X23=31.4`)
	σ_{13} = 24.89 MPa	(`X13=24.89`)
	σ_{12} = 24.89 MPa	(`X12=24.89`)

Data From Avg. Material Allowables for Composite

Failure stresses:	σ_{11} = 2112. MPa	(`X11=2112.`)
	σ_{22} = 84.16 MPa	(`X22=84.16`)
	σ_{33} = 84.16 MPa	(`X33=84.16`)
	σ_{23} = 31.4 MPa	(`X23=31.4`)
	σ_{13} = 57.64 MPa	(`X13=57.64`)
	σ_{12} = 57.64 MPa	(`X12=57.64`)
Action to take upon failure:	Stop execution	(`ACTION=-1`)

The RUC level failure analysis input data is similar to the input data for subcell static failure analysis. As with the subcell static failure capabilities, MAC/GMC 4.0 includes the maximum stress and strain criteria, as well as the Tsai-Hill criterion within its RUC static failure capabilities. In this example problem, two sets of RUC static failure parameters are included within the input file (both of which are commented in the input file above). These two sets of data were taken from the allowables estimated for the unidirectional graphite/epoxy composite (which occupies each of the seven plies in the present example) in Example Problem 5a. One set corresponds to the 1st subcell failure allowable stresses, while the other corresponds to the average material failure allowable stresses.

In order to execute the code to generate all of the results shown below, the appropriate lines in the input file must be commented and uncommented such that only one RUC static failure criterion is active with the subcell failure criteria inactive or such that only the subcell failure criteria are active.

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

Print level:	6	(`NPL=6`)

b) x-y plots (**`*XYPLOT`**): None

7) End of file keyword: (**`*END`**)

Results

For the case that utilizes the subcell static failure criterion and identical σ_{11} and σ_{22} mechanical loading rates (as shown in the input file above), the laminate reaches 1st subcell failure at an applied force resultant level of $N_{xx} = N_{yy} = 774$ MPa × thickness. At this point the code stops execution in accordance with the specified `ACTION=-1`, and the following is written to the output file:

```
*******  SUBCELL FAILURE  ******
  LAYER # 1    INTEGRATION POINT #1
  SUBCELL #  325    MATERIAL #  1
  FAILED DUE TO THE TSAI-HILL CRITERION
  STRESSES:    0.2470E+04   0.6041E+02  -0.4911E+01
               0.0000E+00   0.0000E+00   0.1427E-07
  TSAI-HILL FUNCTION =     1.00000697203627
  ALLOWABLE FUNCTION =     1.00000000000000
  ACTION = -1 ----> STOP EXECUTION
```

As stated above, by altering the ratio of the applied N_{xx} and N_{yy} loading, the "failure surfaces" shown in Figure 5.4 can be generated. To generate the three distinct failure surfaces, the failure simulation is switched between the two sets of RUC failure parameters (1st Cell Failure surfaces) and the subcell failure criteria (1st Subcell Failure surfaces). Figure 5.4 shows that localizing all the way to the level of the subcells within each ply of the laminate in order to generate the 1st Subcell Failure surface for the laminate gives very similar results to the surface generated by localizing only to the level of the RUC within each ply and using the previously generated 1st subcell allowables for the RUC failure stresses. Employing the average material allowables for the RUC failure stresses, however, produces a much larger (i.e., less conservative) laminate failure surface.

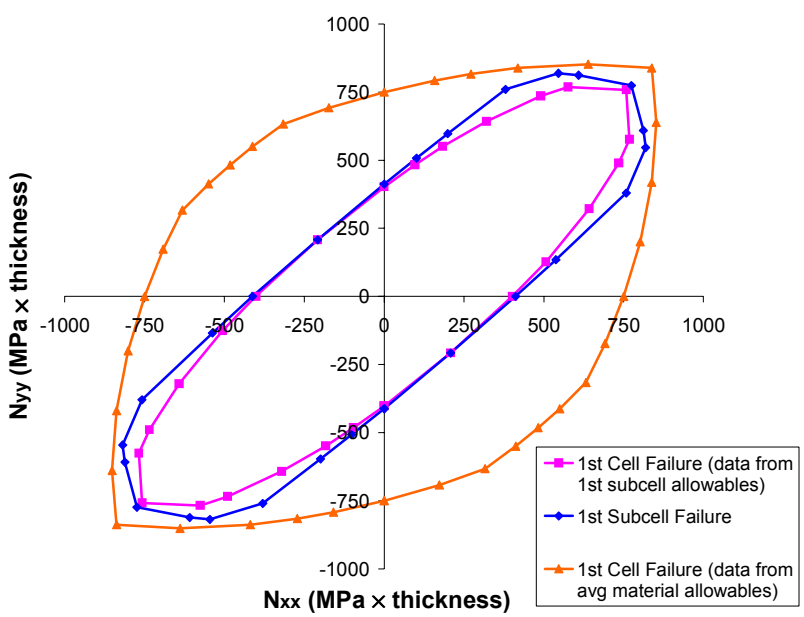

Figure 5.4 Example 5c: Predicted failure surfaces for a 60% fiber volume fraction quasi-isotropic graphite/epoxy laminate at room temperature. Note that these failure surfaces were generated by manually adjusting the ratio of the applied N_{xx} and N_{yy}.