Example 4f: RUC Yield Surface Analysis

This example problem demonstrates one of the more powerful capabilities of MAC/GMC 4.0, the generation of yield surfaces. A yield surface is defined as the locus of points in a stress space at which a specified "yield" condition is satisfied. For metals, a 0.1 % or 0.2 % permanent (inelastic) strain is often used as a yield condition, for example. However, this type of condition is based on uniaxial tension or compression, whereas, a yield surface, by definition is concerned with a multi-axial state of stress. A typical yield surface for a monolithic metal (aluminum, in this case) is shown in Figure 4.9. The "stress space" for this yield surface is the σ_{11}-σ_{22} plane defined by the axes. The plotted yield surface then represents σ_{11}-σ_{22} points that cause the metal to yield. Of course, this implies that some definition of what constitutes yielding has been employed. In Figure 4.9, yielding was defined as an equivalent plastic strain (EPS) of 0.001 (0.1 %), where $EPS = \sqrt{\frac{2}{3}\varepsilon_{ij}^{p}\varepsilon_{ij}^{p}}$ and ε_{ij}^{p} are the plastic strain components. Since EPS is a scalar value based on the multi-axial plastic strain state, it is suitable for use as a yield condition. The elliptical shape of the yield surface indicates that it is based on a von Mises type yield criterion. Indeed, the "yield surface" in Figure 4.9 was generated using the Bodner-Partom viscoplastic constitutive model, which employs a von Mises (J_2) type flow law. In viscoplasticity theory, no explicit yield condition exists that must be satisfied at all times. Therefore, a "yield" or "threshold" surface can be predicted for a viscoplastic material by specifying a particular criterion and checking for when it is satisfied during applied loading. This type of surface can then be generated for any material, regardless of the inelastic model formulation. However, some yield conditions are better suited than others for use in conjunction with particular inelastic constitutive models. For example, a surface generated from a rate-based criterion would not be appropriate for use in conjunction with a rate independent inelastic model such as incremental plasticity. It should be noted that, in its present form, the yield surface analysis implementation within MAC/GMC 4.0 is most relevant to composites and laminates that contain metals. However, the capabilities could also be applied to polymers and ceramics that exhibit inelastic deformation. For additional information on the code's yield surface predictions, see Section 6 of the Theory Manual.

In order to generate a yield surface like that shown in Figure 4.9, MAC/GMC 4.0 applies combined mechanical loading in the given stress space. That is, loading is applied along a specific angle in the σ_{11}-σ_{22} plane (for the present case) until the yield definition is encountered. Then, a new angle is chosen along which loading is applied until yield. This angle is referred to as the "probing angle". When a sufficient number of angles have been probed, the locus of all the points at which yield occurred defines the yield surface. In MAC/GMC 4.0, the user specifies the increment to employ for the probing angle. The code then probes at angles from 0° to 360° using this angle increment. A small angle increment will generate a smoother yield surface, but requires more execution time as the code must execute for each probing angle.

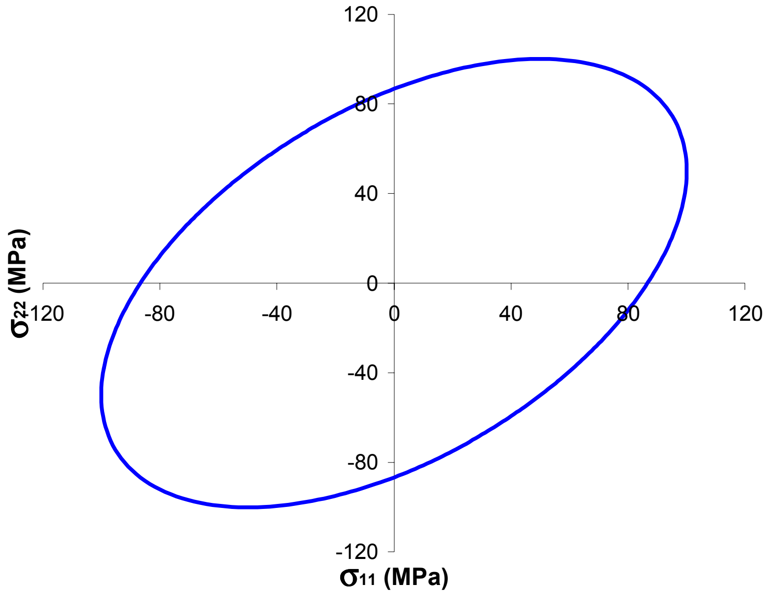

Figure 4.9 Yield surface for aluminum at room temperature based on EPS=0.001.

MAC/GMC Input File: `example_4f.mac`

```
MAC/GMC 4.0 Example 4f - RUC yield surface analysis
*CONSTITUENTS
  NMATS=2
  M=1 CMOD=6 MATID=E
  M=2 CMOD=4 MATID=A
*RUC
  MOD=2 ARCHID=1 VF=0.25 F=1 M=2
#*THERM
#  NPT=3 TI=0.,57600.,64800. TEMP=900.,23.,650.
#*SOLVER
#  METHOD=1 NPT=3 TI=0.,57600.,64800. STP=40.,40.
*SURF
  TMAX=400. STP=0.01 MMAX=0.04 MODE=1 TREF=650.
  OPTION=1,2,3,4
  ISPX=1 ISPY=2 ANGINC=5.
  EPS=0.0001 DR=0.00025 ISR=0.00002 IP=0.001
*PRINT
  NPL=1
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (**`*CONSTITUENTS`**) [KM_2]:

Number of materials:	2	(`NMATS=2`)
Materials:	SiC fiber	(`MATID=E`)
	Ti-21S	(`MATID=A`)

Constitutive models:	SiC fiber: linearly elastic	(CMOD=6)
	Ti-21S matrix: Isotropic GVIPS	(CMOD=4)

3) Analysis type (***RUC**) → Repeating Unit Cell Analysis [KM_3]:

Analysis model:	Doubly periodic GMC	(MOD=2)
RUC architecture:	Square fiber, square pack	(ARCHID=1)
Fiber volume fraction:	0.25	(VF=0.25)
Material assignment:	SiC fiber	(F=1)
	Ti-21S matrix	(M=2)

4) Loading:

a) Mechanical (***MECH**): None

The loading for yield surface probing is controlled under ***SURF**. Any specified mechanical or thermal loading is interpreted as "preloading", which is applied prior to beginning the yield surface probing. This allows the user to incorporate a certain stress/strain state into the composite prior to determination of the yield surface.

b) Thermal (***THERM**) [KM_4]: (case with residual stresses only)

Number of points:	3	(NPT=3)
Time points:	0., 57600., 64800. sec.	(TI=0.,57600.,64800.)
Temperature points:	900., 23., 650.	(TEMP=900.,23.,650.)

c) Time integration (***SOLVER**) [KM_4]: (case with residual stresses only)

Time integration method:	Forward Euler	(METHOD=1)
Number of points:	3	(NPT=3)
Time points:	0., 57600., 64800. sec.	(TI=0.,57600.,64800.)
Time step size:	40., 40. sec.	(STP=40.,40.)

☞ Note: As with the mechanical loading, the thermal loading in yield surface analysis is interpreted by the code as "preloading". Thus, the thermal loading specified allows residual stresses to be included in the composite prior to the yield surface probing. To execute this example problem without residual stresses, the ***THERM** and ***SOLVER** keyword data should be commented as shown above. To execute with residual stresses, these should be uncommented.

d) Yield surface generation (***SURF**) [KM_4]:

Maximum probing time:	400. sec.	(TMAX=400.)
Probing time step size:	0.01 sec.	(STP=0.01)
Maximum probing load:	0.04	(MMAX=0.04)
Probing mode:	strain control	(MODE=1)
Reference temperature:	650. °C	(TREF=650.)
Yield surface options:	Generate global data (1)	(OPTION=1,2,3,4)
	Generate 1st subcell data (2)	
	Generate all subcells data (3)	
	Generate local subcell data (4)	

Stress space x-axis:	σ_{11}	(`ISPX=1`)
Stress space y-axis:	σ_{22}	(`ISPY=2`)
Probe angle increment:	5.°	(`ANGINC=5`)
Quantities defining yield:	Equivalent plastic strain = 0.0001	(`EPS=0.0001`)
	Dissipation rate = 0.00025	(`DR=0.00025`)
	Inelastic strain rate = 0.00002	(`ISR=0.00002`)
	Inelastic power = 0.001	(`IP=0.001`)

The mechanical loading for the yield surface probing is specified under **`*SURF`** in the form of a maximum time (`TMAX`), time step (`STP`), maximum load (`MMAX`), and mode (`MODE`). The yield surface mechanical loading is unique from that specified under **`*MECH`** in that only one segment may be employed. If, at a particular probing angle, the maximum time is reached before all yield criteria are satisfied, a warning is written to the output file, and the probing proceeds to the next angle. Thus a large maximum probing load and maximum probing time should be employed. In addition, the time step size needs to be reasonably small such that the exact point of yielding can be captured. The rate of the applied loading is determined from the maximum time and maximum load data. This rate is interpreted as a J_2 (stress) or J_2' (strain) quantity. This allows the loading rate to be consistent as the probing angle changes. If thermal preloading is not specified with **`*THERM`**, then `TREF` must be specified under **`*SURF`** in order to indicate the temperature at which the yield surface is determined. If thermal preloading is specified, this `TREF` is ignored and the ending temperature from the thermal preloading is employed.

The yield surface option (**`*SURF`**) generates up to four ASCII files (for the case of RUC analysis) that contain the yield surface data. These are: `surf_global.dat`, `surf_1st.dat`, `surf_all.dat`, and `surf_local.dat`. `surf_global.dat` contains the global yield surface data for each type of yield surface (i.e., EPS, DR, ISR, and IP), `surf_1st.dat` contains the yield surface data based on first subcell yield for each type of yield surface, `surf_all.dat` contains the yield surface data based on the requirement that all subcells (that can yield) yield for each type of yield surface, and `surf_local.dat` contains the yield surface data for each individual subcell for each type of yield surface.

The stress space is defined by `ISPX` and `ISPY`. In this example, the σ_{11}-σ_{22} stress space is specified, which indicates that loading will be applied in the x_1-direction (along the fiber direction) and the x_2-direction (transverse to the fiber direction). Additional options are available using `ISPX` and `ISPY`, including applying additional loading components that are slaves to (i.e., equal to) the primary stress space loading components. These options are described in the MAC/GMC 4.0 Keywords Manual Section 4.

The yield surface probing occurs along angles in the stress space determined by the specified probe angle increment (`ANGINC`). That is, for example, when probing at an angle of 5° in the σ_{11}-σ_{22} stress space, the code applies loading such that $\sigma_{11}/\sigma_{22} = \tan(5°)$. Upon fulfillment of all yield criteria (or upon reaching the end of the specified maximum time), the code proceeds to the next probing angle. If preloading is specified, the code begins each probing angle not at a zero stress-strain state, but rather at the stress-strain state that existed at the end of the preload. A smaller probe angle increment results in a smoother predicted yield surface, but will increase the execution time. For additional information on the MAC/GMC 4.0 yield surface capabilities and procedures, see the MAC/GMC 4.0 Keywords Manual Section 4.

The particular yield values chosen in this example problem are arbitrary. It is possible to see when yielding occurs at a particular probe angle by generating an x-y plot and obtaining a particular yield point from the notification written to the output file. The yield values can the be adjusted as desired to make the yield prediction more or less conservative.

5) Damage and Failure: None

6) Output:
  a) Output file print level (***PRINT**) [KM_6]:
    Print level: 0 (NPL=0)

  b) x-y plots (***XYPLOT**): None

7) End of file keyword: (***END**)

Results

Some of the yield surface data results from this example are plotted in Figure 4.10 – Figure 4.15. All yield surface data generated have not been plotted as this example generates data for six yield surfaces of each type (i.e., EPS, DR, ISR, and IP) for both the cases with and without residual. Figure 4.10 shows the global yield surfaces for the case without residual stresses. The term “global” refers to the fact that the composite or RUC level stresses, strains, and rates have been employed to determine when yielding occurs. All four yield surfaces in Figure 4.10 are biased (i.e., elongated) along the σ_{11}-axis due to the presence of the continuous fibers, which elevate the yield stress, in the x_1-direction. Based on the yield values specified in the input files (which are somewhat arbitrary), the composite tends to yield first with respect to the dissipation rate (DR), followed by the inelastic power (IP), the inelastic strain rate (ISR), and finally by the equivalent plastic strain (EPS). The four yield surfaces are similar in shape and all are centered about the stress space origin, with the ISR surface being the most elongated.

Figure 4.11 shows the same four yield surfaces for the case in which residual stresses have been incorporated. Qualitatively, these yield surfaces are similar to their counterparts from Figure 4.10. The major difference is that the yield surfaces are shifted to the left due to the compressive residual stresses in the matrix.

Figure 4.12 presents all six equivalent plastic strain (EPS) yield surfaces from the case in which residual stresses were not included. The global yield surface is the same as that plotted in Figure 4.10 (labeled “EPS Global”). Also plotted are the yield surfaces associated with the three individual matrix subcells (β,γ = 1,2; 2,1; and 2,2), see Figure 4.16. Because the 2,2 subcell is farthest from the fiber, this subcell has a significantly larger yield surface than the 1,2 and 2,1 subcells. The 1,1 subcell is occupied by the elastic fiber and thus does not have a yield surface. The first subcell yield surface generated by MAC/GMC 4.0 traces the intersection of the individual subcell yield surfaces. Conversely the yield surface generated by requiring all subcells to yield traces the union of the individual subcell yield surfaces (which, in Figure 4.12, coincides with the subcell 2,2 yield surface).

The counterpart to Figure 4.12 is Figure 4.13, which contains the same six yield surfaces for the case including residual stresses. Now, the subcell 2,2 yield surface intersects the subcell 1,2 and 2,1 yield surfaces, giving rise to irregularly shaped first subcell and all subcells yield surfaces in the vicinity of the tensile σ_{11}-axis. This irregularity of the all subcells EPS yield surface can be seen more clearly in Figure 4.14, which directly compares EPS yield surfaces for the cases with and without residual stresses. Further, this figure shows that residual stresses have translated the yield surfaces slightly in the direction of the tensile σ_{22}-axis in addition to the compressive σ_{11} translation mentioned previously. Finally, Figure 4.15 compares inelastic strain rate (ISR) yield surfaces for the cases with and without residual stresses.

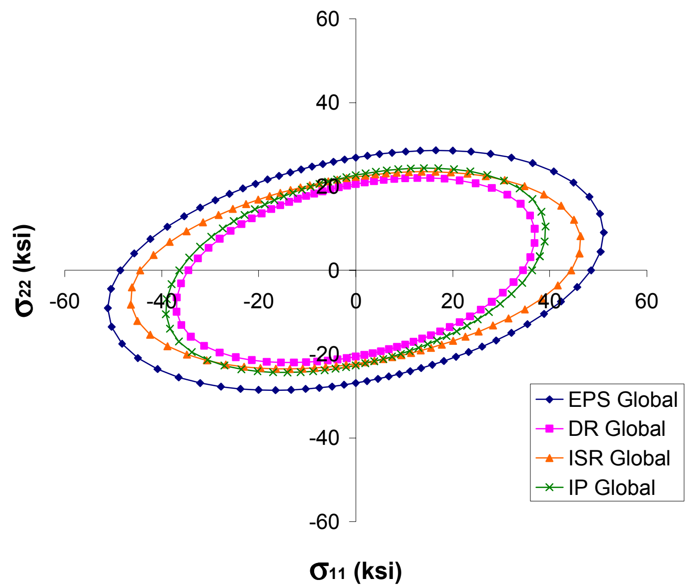

Figure 4.10 Global (RUC) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with no residual stresses.

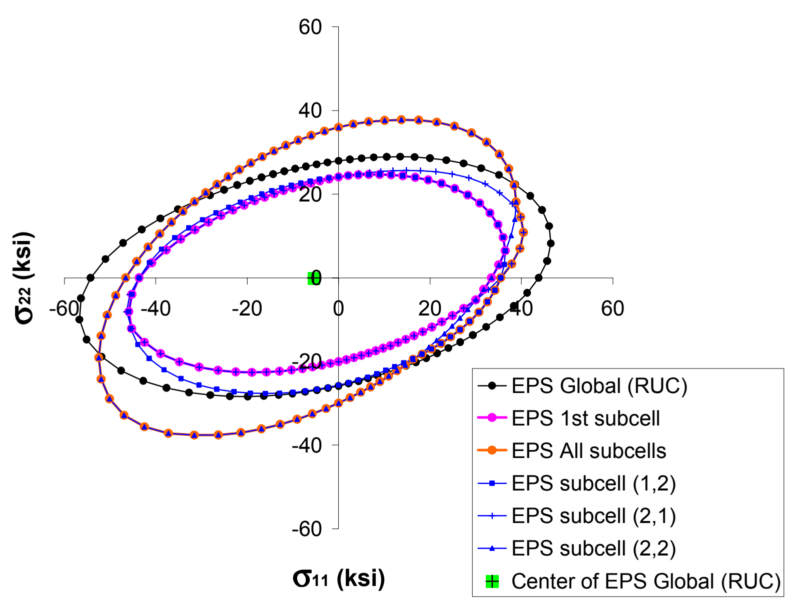

Figure 4.11 Global (RUC) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with residual stresses.

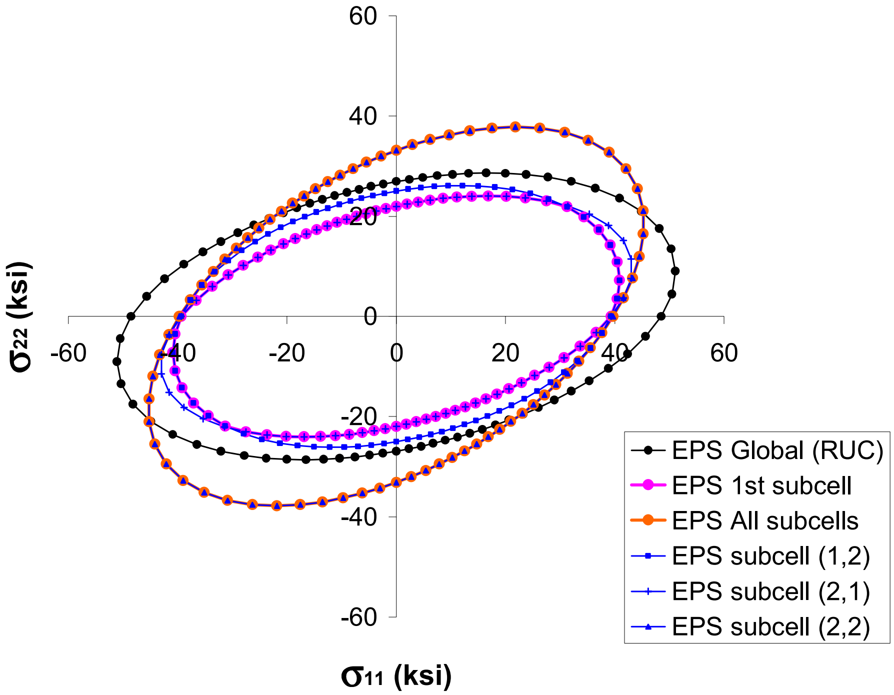

Figure 4.12 Local and global equivalent plastic strain (EPS) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with no residual stresses.

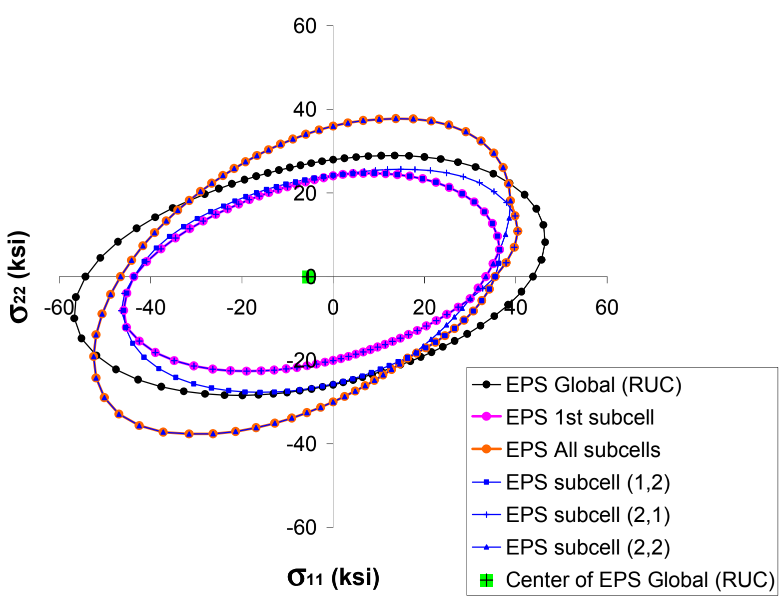

Figure 4.13 Local and global equivalent plastic strain (EPS) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with residual stresses.

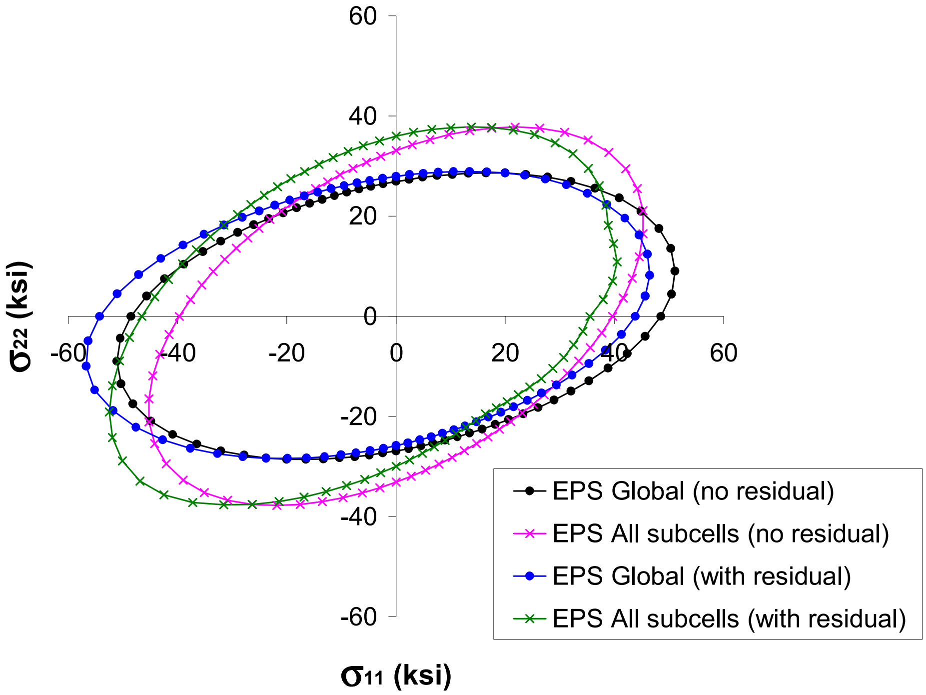

Figure 4.14 Comparison of equivalent plastic strain (EPS) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with and without residual stresses.

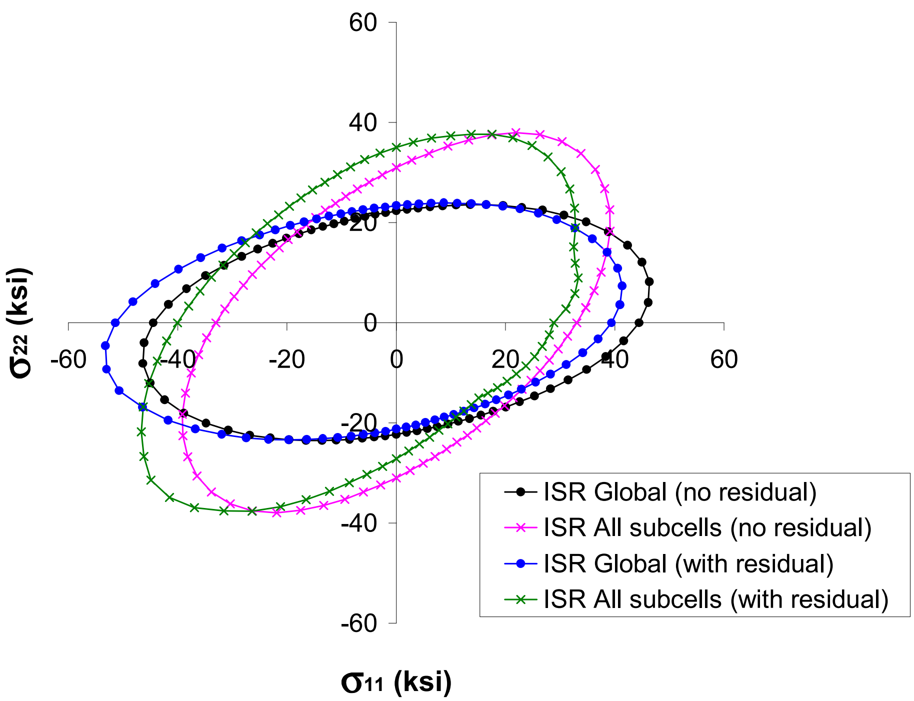

Figure 4.15 Comparison of inelastic strain rate (ISR) yield surfaces for 0.25 fiber volume fraction SiC/Ti-21S at 650 °C with and without residual stresses.

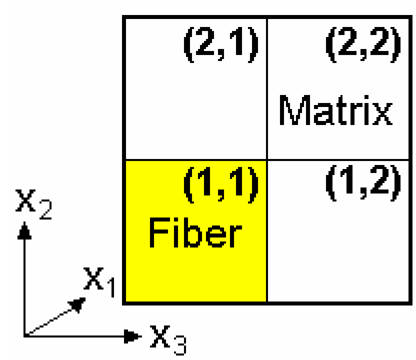

Figure 4.16 2×2 RUC representing the 0.25 fiber volume fraction SiC/Ti-21S composite in Example 4f. The subcell indices (β,γ) are indicated.