Example 3h: Cross-Ply and Quasi-Isotropic Laminates

This example problem employs the classical lamination theory capabilities of MAC/GMC 4.0 to simulate the response of two SiC/Ti-21S laminates: cross-ply $[90°/0°]_s$ and quasi-isotropic $[90°/45°/0°/-45°]_s$. As in Example 3g, a midplane strain of 0.02 is applied to the laminate, however, in the present problem a more refined RUC architecture is employed for each layer. For more information on the lamination theory and the code's laminate analysis capabilities, see the MAC/GMC 4.0 Theory Manual Section 3.

MAC/GMC Input File: **example_3h.mac**

```
MAC/GMC 4.0 Example 3h - Cross-ply and quasi-isotropic laminates
*CONSTITUENTS
  NMATS=2
  M=1 CMOD=6 MATID=E
  M=2 CMOD=4 MATID=A
*LAMINATE
# -- Cross-ply
  NLY=3
  LY=1 MOD=2 THK=0.25 ANG=90 ARCHID=6 R=1. VF=0.25 F=1 M=2
  LY=2 MOD=2 THK=0.50 ANG=0  ARCHID=6 R=1. VF=0.25 F=1 M=2
  LY=3 MOD=2 THK=0.25 ANG=90 ARCHID=6 R=1. VF=0.25 F=1 M=2
# -- Quasi-isotropic
#  NLY=7
#  LY=1 MOD=2 THK=0.125 ANG=90  ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=2 MOD=2 THK=0.125 ANG=45  ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=3 MOD=2 THK=0.125 ANG=0   ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=4 MOD=2 THK=0.25  ANG=-45 ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=5 MOD=2 THK=0.125 ANG=0   ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=6 MOD=2 THK=0.125 ANG=45  ARCHID=6 R=1. VF=0.25 F=1 M=2
#  LY=7 MOD=2 THK=0.125 ANG=90  ARCHID=6 R=1. VF=0.25 F=1 M=2
*MECH
  LOP=1
  NPT=2 TI=0.,200. MAG=0.,0.02 MODE=1
*THERM
  NPT=2 TI=0.,200. TEMP=650.,650.
*SOLVER
  METHOD=1 NPT=2 TI=0.,200. STP=1.
*PRINT
  NPL=6
*XYPLOT
  FREQ=5
  LAMINATE=1
   NAME=example_3h X=1 Y=10
  MACRO=0
  MICRO=0
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (**`*CONSTITUENTS`**) [KM_2]:

Number of materials:	2	(`NMATS=2`)
Materials:	SiC fiber	(`MATID=E`)
	Ti-21S	(`MATID=A`)
Constitutive models:	SiC fiber: linearly elastic	(`CMOD=6`)
	Ti-21S matrix: Isotropic GVIPS	(`CMOD=4`)

3) Analysis type (**`*LAMINATE`**) → Laminate Analysis [KM_3]:

Cross-Ply Laminate:
Number of layers: 3 (`NLY=3`)

Layer	**Analysis Model**	**Thickness**	**Fiber Angle**	**Architecture**	**Aspect Ratio**	**Volume fraction**	**Fiber material**	**Matrix material**
(`LY=`)	(`MOD`)	(`THK`)	(`ANG`)	(`ARCHID`)	(`R`)	(`VF`)	(`F`)	(`M`)
1	GMC-2D	0.25	90°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
2	GMC-2D	0.50	0°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
3	GMC-2D	0.25	90°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S

Quasi-Isotropic Laminate:
Number of layers: 7 (`NLY=7`)

Layer	**Analysis Model**	**Thickness**	**Fiber Angle**	**Architecture**	**Aspect Ratio**	**Volume fraction**	**Fiber material**	**Matrix material**
(`LY=`)	(`MOD=2`)	(`THK=`)	(`ANG=`)	(`ARCHID=6`)	(`R=1.`)	(`VF=0.25`)	(`F=1`)	(`M=2`)
1	GMC-2D	0.125	90°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
2	GMC-2D	0.125	45°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
3	GMC-2D	0.125	0°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
4	GMC-2D	0.25	-45°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
5	GMC-2D	0.125	0°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
6	GMC-2D	0.125	45°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S
7	GMC-2D	0.125	90°	7×7 circle, rect. pack	1.	0.25	SiC	Ti-21S

☞ Note: In order to execute the code for the two different laminates, the appropriate lines in the input file must be commented and uncommented. For more information on the laminate analysis input requirements, see the MAC/GMC 4.0 Keywords Manual Section 3.

☞ Note: The middle layers of both laminates are twice as thick as the other layers because these layers actually represent two individual layers that are adjacent, and thus can be combined in the analysis.

4) Loading:

a) Mechanical (**`*MECH`**) [KM_4]:

Loading option:	1 (loading in the laminate x-direction)	`(LOP=1)`
Number of points:	2	`(NPT=2)`
Time points:	0., 200. sec.	`(TI=0.,200.)`
Load magnitude:	0., 0.02	`(MAG=0.,0.02)`
Loading mode:	midplane strain/curvature control	`(MODE=1)`

b) Thermal (**`*THERM`**) [KM_4]:

Number of points:	2	`(NPT=2)`
Time points:	0., 200. sec.	`(TI=0.,200.)`
Temperature points:	650., 650. °C	`(TEMP=650.,650.)`

c) Time integration (**`*SOLVER`**) [KM_4]:

Time integration method:	Forward Euler	`(METHOD=1)`
Number of points:	2	`(NPT=2)`
Time points:	0., 200. sec.	`(TI=0.,200.)`
Time step sizes:	1. sec.	`(STP=1.)`

5) Damage and Failure: None

6) Output:

a) Output file print level (**`*PRINT`**) [KM_6]:

Print level:	6	`(NPL=6)`

b) x-y plots (**`*XYPLOT`**) [KM_6]:

Frequency:	5	`(FREQ=5)`
Number of laminate plots:	1	`(LAMINATE=1)`
Laminate plot name:	example_3h	`(NAME=example_3h)`
Laminate plot x-y quantities:	ε^0_{xx} , N_{xx}	`(X=1 Y=10)`
Number of macro plots:	0	`(MACRO=0)`
Number of micro plots:	0	`(MICRO=0)`

7) End of file keyword: (**`*END`**)

Results

Table 3.1 and Figure 3.17 show that the cross-ply laminate exhibits a stiffer global response than does the quasi-isotropic laminate. The difference is especially clear in the post-yield response depicted in Figure 3.17. In addition, from Table 3.1, it is clear that neither laminate exhibits global extensional normal-shear coupling as the terms A_{16} and A_{26} are zero (or nearly zero) in both cases. However, since the terms D_{16} and D_{26} are non-zero for the quasi-isotropic laminate, normal-shear bending coupling is present, whereas this coupling is absent in the cross-ply laminate. Also plotted in Figure 3.17 is the response of a

0.25 fiber volume fraction SiC/Ti-21S composite whose geometry is represented by a cross-ply continuum architecture (see Figure 3.10) using triply periodic GMC from Example 3e. For the present case, the continuum and lamination theory simulations are in good agreement. Finally, examining the effective laminate engineering constants in Table 3.1, it is clear why the [90°/45°/0°/-45$^\circ_s$] laminate is termed "quasi-isotropic", while the [90°/0°]$_s$ laminate is not. For isotropic materials, the relation among the elastic engineering constants is,

$$G = \frac{E}{2\,(1+\nu)}$$

Substituting the effective laminate E and ν engineering constants for the quasi-isotropic laminate results in a calculated isotropic shear modulus value of $G = 6650.$ ksi, which agrees with the value determined for the laminate given in Table 3.1. In contrast, this value calculated for the cross-ply laminate is $G = 7596.$ ksi, which does not agree with the value determined for the laminate in Table 3.1. Thus, the [90°/45°/0°/-45°]s laminate exhibits "quasi-isotropy" in its effective engineering constants, while the [90°/0°]s laminate does not.

Table 3.1 Example 3h: effective stiffness and engineering constant results for the 0.25 volume fraction SiC/Ti-21S laminates analyzed taken from the MAC/GMC 4.0 output file.

Cross-Ply	**Quasi-Isotropic**
Laminate Axial Stiffness Matrix [A]	Laminate Axial Stiffness Matrix [A]
2.130E+04 **6.108E+03** 7.791E-08 **6.108E+03** **2.130E+04** -6.978E-07 7.791E-08 -6.978E-07 **5.704E+03**	**2.035E+04** **7.054E+03** 3.896E-08 **7.054E+03** **2.035E+04** -3.489E-07 3.896E-08 -3.489E-07 **6.650E+03**
Laminate Coupling Stiffness Matrix [B]	Laminate Coupling Stiffness Matrix [B]
0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00 0.000E+00	0.000E+00 5.684E-14 -2.344E-15 5.684E-14 0.000E+00 1.556E-15 -2.344E-15 1.556E-15 -1.137E-13
Laminate Bending Stiffness Matrix [D]	Laminate Bending Stiffness Matrix [D]
1.586E+03 **5.090E+02** 1.136E-08 **5.090E+02** **1.964E+03** -1.018E-07 1.136E-08 -1.018E-07 **4.754E+02**	**1.608E+03** **5.583E+02** **3.542E+01** **5.583E+02** **1.844E+03** **3.542E+01** **3.542E+01** **3.542E+01** **5.246E+02**
Laminate Engineering Constants	Laminate Engineering Constants
E11= 1.955E+04 N12= 2.868E-01 E22= 1.955E+04 G12= 5.704E+03	E11= 1.791E+04 N12= 3.466E-01 E22= 1.791E+04 G12= 6.650E+03

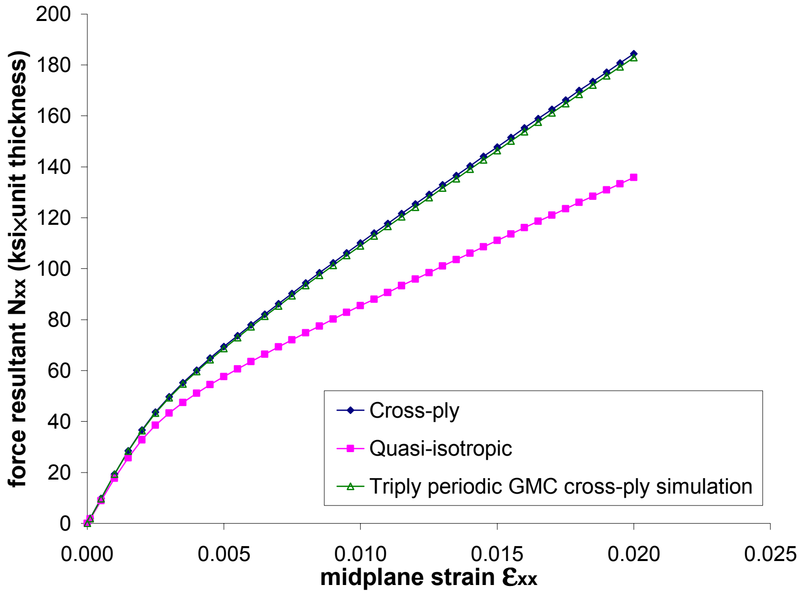

Figure 3.17 Example 3h: plot of the global tensile force resultant – midplane strain ($N_{xx} - \varepsilon_{xx}^{0}$) response for 0.25 fiber volume fraction cross-ply $[90°/0°]_s$ and quasi-isotropic $[90°/45°/0°/-45°]_s$ SiC/Ti-21S laminates at 650 °C. Also plotted is a triply periodic GMC prediction using an RUC like that employed in Example 3e that simulates a cross-ply architecture.