

3.2 Integration Algorithms

There are two integration algorithms currently available within **MAC/GMC**. The first is the standard, explicit Forward Euler algorithm, which can be expressed as,

$$w_{i+1} = w_i + \Delta t f(t_i, w_i)$$

where

$$\begin{aligned} w_i &= \underline{y}(t_i) \\ w_{i+1} &= \underline{y}(t_{i+1}) \end{aligned}$$

and $f(t_i, w_i)$ is the rate of change with respect to time of the vector, $\underline{y}(t_i)$, i.e.

$$f(t_i, w_i) = \dot{\underline{y}} = \frac{d\underline{y}(t_i)}{dt}$$

The second is a predictor/corrector algorithm with a self-adaptive time step which uses:

1) a 4th order Runge-Kutta starter:

$$w_i = w_{i-1} + \frac{(K_1 + 2K_2 + 2K_3 + K_4)}{6}$$

where

$$\begin{aligned} K_1 &= \Delta t f(t_{i-1}, w_{i-1}) \\ K_2 &= \Delta t f\left(t_{i-1} + \frac{\Delta t}{2}, w_{i-1} + \frac{K_1}{2}\right) \\ K_3 &= \Delta t f\left(t_{i-1} + \frac{\Delta t}{2}, w_{i-1} + \frac{K_2}{2}\right) \\ K_4 &= \Delta t f(t_{i-1} + \Delta t, w_{i-1} + K_3) \end{aligned}$$

2) with an Adams-Bashforth four-step predictor:

$$\begin{aligned} w_0 &= \alpha_1 & w_1 &= \alpha_2 & w_2 &= \alpha_3 & w_3 &= \alpha_4 \\ w_{i+1}^P &= w_i + \frac{\Delta t}{24} [55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) \\ &\quad - 9f(t_{i-3}, w_{i-3})] \end{aligned}$$

and 3) an Adams-Moulton four step corrector:

$$w_{i+1}^c = w_i + \frac{\Delta t}{24} [9f(t_{i+1}, w_{i+1}^p) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})]$$

where the α 's come from the 4th order Runge-Kutta starter. Further details may be found in [8].

☞ **Note:** It has been found, based on experience, that for relatively rapid monotonic or cyclic loadings it may be more efficient to use the Forward Euler integrator since the predictor/corrector requires 5 evaluations per step, as shown above. However, in the case of creep, relaxation or slow monotonic or cyclic loading histories, significant increases in solution speeds can be obtained using the predictor/corrector algorithm.

Finally, within **MAC/GMC** the vector \underline{w} as used above contains the following macro quantities:

| <u>position</u> | <u>contents</u> |
|-----------------|--|
| 1 - 6 | Macro Total Strain |
| 7 - 12 | Macro Stress |
| 13 - 18 | Macro Inelastic Strain |
| 19 - 30 | (currently empty space for 2 6x1 vectors) |
| 31 - 36 | Macro Thermal Strain |
| 37 | Current Temperature |
| 38- 46 | Future Use |

and the quantities associated with each subcell are stored sequentially in \underline{w} , such that

| <u>position</u> | <u>contents</u> |
|-----------------|---|
| 47 - 52 | Micro Total Strain |
| 53 - 58 | Micro Stress |
| 59 - 64 | Micro Inelastic Strain |
| 65 - 76 | Micro Internal State Variables (space for 2 6x1 vectors) |
| 77 - 82 | Micro Thermal Strain |
| 83 - 88 | Debonding Parameters |

The above 42 positions (i.e., positions 47-88) are repeated for the total number of subcells (N) thus bringing the total length of the \underline{w} vector to 46+42N. It follows that a second vector of similar length that contains the corresponding macro and micro rates, e.g., $f(t_p, w_i)$, is also utilized.