3.5 Fatigue Damage Analysis

The fatigue damage calculations utilize a recently developed multiaxial, isothermal, continuum damage mechanics model for the fatigue of unidirectional metal matrix composites [10]. The model is phenomenological, stress based, and assumes a single scalar internal damage variable, D. Note that for an initially anisotropic material, the evolution of the damage, although a scalar, is directionally dependent. As will be shown, this directional dependence is accounted for in the terms, $\hat{F}_m$, Φ_{fl}, and Φ_u. The present multiaxial, isothermal, continuum damage model for **initially transversely isotropic materials** (e.g., unidirectional metal matrix composites) may be expressed as, [10]

$$\int_{D_{k-1}}^{D_k} dD = \int_0^N [1-(1-D)^{\beta+1}]^{\alpha}\left[\frac{\hat{F}_m}{1-D}\right]^{\beta} dN \qquad \text{(EQ 29)}$$

where N is the number of cycles at the current stress state, (σ_k), and D_k and D_{k-1} are the amount of damage at the current and previous increments, respectively. The quantity α which is a function of the current stress state is defined as,

$$\alpha = 1 - a\frac{\langle\Phi_{fl}\rangle}{\langle\Phi_u\rangle} \qquad \text{(EQ 30)}$$

where $\langle\ \rangle$ are the Macauley brackets. In the above, the fatigue limit surface, Φ_{fl}, and the static fracture surface, Φ_u, are defined as

$$\Phi_{fl} = \frac{1}{2}\frac{max}{t_0}\frac{max}{t}\ F_{(\sigma_{fl})}(\sigma_{ij}(t)-\sigma_{ij}(t_0)) - 1 \qquad \text{(EQ 31)}$$

$$\Phi_u = 1 - \frac{max}{t}\ F_{(\sigma_u)}(\sigma_{ij}(t)) \qquad \text{(EQ 32)}$$

and the quantity $\hat{F}_m$, used in EQ. 29 is the normalized stress amplitude, and is defined as,

$$\hat{F}_m = \frac{1}{2}\frac{max}{t}\frac{max}{t_0}\ F_{(M)}(\sigma_{ij}(t)-\sigma_{ij}(t_0)) \qquad \text{(EQ 33)}$$

Note, the case $\langle\Phi_u\rangle = 0$ indicates static fracture, which is failure, making it unnecessary to perform the fatigue calculations as in this case the subcell is considered to have failed completely. Thus, having to consider the possibility of α being undefined is unnecessary. The case $\langle\Phi_{fl}\rangle = 0$ indicates that the current stress state is below the fatigue limit and thus α is set equal to 1. This presents a special case when integrating the fatigue damage expression, EQ. 29, and will be considered later in this section.

In the above equations, t_0 is the time at the beginning of the current load cycle, and t, is some time during the load cycle. The general form for $F_{(\sigma_{fl}),\ (\sigma_u),\ or\ (M)}$ may be expressed as,

$$F_{(\ )} = \sqrt{\frac{1}{(\ )_L^2}\left\{(4\omega_{(\ )}^2 - 1)I_1 + \frac{4\omega_{(\ )}^2 - 1}{\eta_{(\ )}^2}I_2 + \frac{9}{4}I_3\right\}} \quad \text{(EQ 34)}$$

It is in the above expression, in which the evolution of the damage becomes directionally dependent. This simply amounts to the assumption of partial anisotropy, where the “extent” (magnitude) of damage is affected by the directionality of the stress state. Specifically, the directional dependence enters through the quantities, I_1, I_2, I_3, $\omega_{(\ )}$, and $\eta_{(\ )}$. The quantities, I_1, I_2, I_3 are invariants having the form,

$$\begin{aligned} I_1 &= \frac{1}{2}S_{ij}S_{ij} - d_i d_j S_{jk}S_{ki} + \frac{1}{4}(d_i d_j S_{ij})^2 \\ I_2 &= d_i d_j S_{jk}S_{ki} - (d_i d_j S_{ij})^2 \\ I_3 &= (d_i d_j S_{ij})^2 \end{aligned} \quad \text{(EQ 35)}$$

which are a function of the current deviatoric stress state, $S_{ij}^k = \sigma_{ij}^k - \frac{1}{3}\sigma_{mm}^k\delta_{ij}$, as well as the vector d_i which defines the materials’ preferred direction (e.g., fiber orientation in a composite). In addition, the terms $\omega_{(\ )}$ and $\eta_{(\ )}$ represent the ratios of longitudinal to transverse normal and shear stresses, respectively. Note, the longitudinal direction is parallel to the preferred direction and transverse is perpendicular to the preferred direction. For initially transversely isotropic materials, $\omega_{(\ )}$ and $\eta_{(\ )}$ are > 1 and for isotropic materials $\omega_{(\ )}$ and $\eta_{(\ )}$ are = 1.

In the context of micromechanics analysis within **MAC/GMC**, the isotropic simplification of the above representation will be predominately used for the various constituent phases [11]. This isotropic representation is the previously validated NonLinear Cumulative Damage Rule (NLCDR) developed at **ONERA** (Office Nationale d’Etudes et de Recherches Aerospatiales) for isotropic monolithic metals. However, it maybe desirable to use the transverse isotropic form when dealing with fiber tows in woven composites systems.

3.5.1 Above Initial Fatigue Limit

Given a current state of stress, σ_k, above the fatigue limit, i.e. $\alpha \neq 1$ and integrating EQ. 29 results in an expression for the number of cycles, N, i.e.,

$$N = \frac{([1-(1-D_k)^{\beta+1}]^{1-\alpha} - [1-(1-D_{k-1})^{\beta+1}]^{1-\alpha})}{\hat{F}_m^{\beta}(1-\alpha)(\beta+1)} \qquad \text{(EQ 36)}$$

Note that D_{k-1} is the total amount of damage at the beginning of the load block and D_k is the total amount of damage at the end of this load block. Alternatively, rewriting EQ. 36 an expression for the damage, D_k, in terms of the number of cycles and previous damage can be obtained, i.e.,

$$D_k = 1 - \left(1 - \left\{[1-(1-D_{k-1})^{\beta+1}]^{1-\alpha} \quad + (1-\alpha)(\beta+1)\hat{F}_m^{\beta} N\right\}^{\frac{1}{1-\alpha}}\right)^{\frac{1}{\beta+1}} \qquad \text{(EQ 37)}$$

In the present computational scheme, since the damage increment is controlled, both D_k and D_{k-1} are known. That is, $D_k = D_{k-1} + \Delta D$ where ΔD is the user specified increment in damage. Thus EQ. 36 is used to predict the increment in the number of cycles for each subcell, N^e, due to the imposed increment in damage.

To calculate the number of cycles to failure, for an initial damage amount, D_{k-1}, let $D_k = 1$, which results in the following,

$$N_F = \frac{(1 - [1-(1-D_{k-1})^{\beta-1}]^{1-\alpha})}{\hat{F}_m^{\beta}(1-\alpha)(\beta+1)} \qquad \text{(EQ 38)}$$

3.5.2 Below Initial Fatigue Limit

Now consider the case in which the current stress state is below the initial fatigue limit, i.e. $\langle \Phi_{fl} \rangle = 0$, which leads to $\alpha_k = 1$. Thus, EQ. 29 takes the form,

$$\int_{D_{k-1}}^{D_k} \frac{(1-D)^{\beta}}{1-(1-D)^{\beta+1}} dD = \int_0^N \hat{F}_m^{\beta} dN \qquad \text{(EQ 39)}$$

Upon integrating the above equation, the increment in cycles, N, with initial damage, D_{k-1}, may be expressed as,

$$N = \left(\frac{\log[1-(1-D_k)^{\beta+1}] - \log[1-(1-D_{k-1})^{\beta+1}]}{\hat{F}_m^{\beta}(\beta+1)}\right) \qquad \text{(EQ 40)}$$

Alternatively, the following expression for the damage, D_k, may be expressed as:

$$D_k = 1 - \{1 - [1 - (1 - D_{k-1})^{\beta + 1}] exp((\beta + 1)\hat{F}_m^{\beta} N)\}^{\frac{1}{\beta + 1}} \quad \text{(EQ 41)}$$

For the number of cycles to failure, let $D_k = 1$,

$$N_F = \frac{-\log[1 - (1 - D_{k-1})^{\beta + 1}]}{\hat{F}_m^{\beta}(\beta + 1)} \quad \text{(EQ 42)}$$

The effect of damage is included in the present micromechanics analysis utilizing the concept of <u>effective stress</u> and the hypothesis of strain-equivalence [12].

$$\hat{\underset{\sim}{\sigma}} = \frac{\underset{\sim}{\sigma}}{(1 - D_k)} \quad \text{(EQ 43)}$$